

Some Clearer Reactions: Gauker on the Validity of Universal Instantiation¹

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Abstract: It is part of the logical orthodoxy that quantifiers are interdefinable and that the rules of Universal Instantiation (UI) and Existential Generalization (EG) hold or fail together. Christopher Gauker has presented some cases which seemingly undermine the validity of UI but nonetheless leave EG untouched, and has developed a very sophisticated theory to explain why this is so. In the process, he has rejected several attempts to explain the asymmetry, especially those aiming at saving the logical orthodoxy by showing what is wrong with the counterexamples to UI. In this paper I argue that some of those proposals are better grounded than Gauker thinks and that ultimately they should be preferred over his since they explain satisfactorily the apparent counterexamples.

Keywords: universal languages; many-sorted and typed languages; logical orthodoxy; ordinary intuitions; contexts instantiation (UI); existential generalization (EG).

1. Introduction

Let us recall some of the logical orthodoxy. Where the horizontal line is read “therefore”, the rules of *universal instantiation* (UI) and *existential generalization* (EG) are:

$$\text{UI: } \frac{\text{Every } x \text{ is } F}{a \text{ is } F} \qquad \text{EG: } \frac{a \text{ is } F}{\text{Some } x \text{ is } F}$$

More formally –where P is a formula and ‘ $P(t/x)$ ’ is the result of substituting a

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term t for each free (non-quantified) occurrence of x in P — and more generally:

$$\text{UI: } \frac{\forall x P}{P(t/x)} \qquad \text{EG: } \frac{P(t/x)}{\exists x P}$$

provided that t is free for x in P . (For simplicity, in what follows I will omit the restrictions when formulating the rules.) The quantifiers are interdefinable using negation, \neg , as follows:

$$\begin{aligned} \forall x P \text{ is equivalent to } & \neg \exists x \neg P \\ \exists x P \text{ is equivalent to } & \neg \forall x \neg P \end{aligned}$$

UI is valid if and only if EG is valid. This can be proven, for example, given the form of the rules, the interdefinability of quantifiers, double negation elimination and a form of contraposition for valid rules. Suppose UI. Then, contraposing:

$$\frac{\neg P(t/x)}{\neg \forall x P}$$

Negating the atomic part both above and below the line:

$$\frac{\neg \neg P(t/x)}{\neg \forall x \neg P}$$

From double negation elimination and the definition of $\exists x$:

$$\frac{P(t/x)}{\exists x P}$$

i.e. EG. The remaining part of the proof is similar to the above and can be left to the reader.

However, UI might seem to have counterexamples. Let us consider the following scenario, adapted from Mion (2014): After a teacher takes attendance in a class, she claims “Everybody is present”. She then applies the rule UI to “Everybody is present” and since ‘Hypatia’ is a name in her language, she infers that Hypatia is present. Were the rule valid and the premise true, then Hypatia would have to be present, but clearly she is not. Nonetheless, the elements of this scenario seemingly pose no problem for EG: If she claims truly in a class “Hypatia is present”, she can conclude “Someone is present” without any problem. This is Christopher Gauker’s (1997, 2003) *asymmetry problem*: Explain why if (1) the quantifiers involved in UI and EG are interdefinable and (2) UI seems to have counterexamples, (3) EG is not subject to the same kind of counterexamples.

Gauker takes the counterexamples at face value and develops a theory in

which the asymmetry emerges quite naturally. In making his case, he discusses and rejects “some confused reactions” to the purported counterexamples of UI. I am going to argue that at least two of those reactions can be presented in stronger ways and that Gauker has wrongly declared them confused and unviable as answers to the asymmetry problem. Moreover, given that they are less theoretically expensive than Gauker’s proposal, they should be preferred over it, although in the end I will favor one of them.

The paper is structured as follows. In section 2, I survey different approaches to the asymmetry problem and rank them according to their degree of departure from both logical orthodoxy and ordinary intuitions. In section 3, I present Gauker’s case for his preferred option to deal with the asymmetry problem and against the other attempts considered. In section 4, I argue for one of the solutions dismissed by Gauker, namely using quantifiers restricted to domains. The word ‘domain’ might seem an illegitimate intrusion of model-theoretic notions in the formulation of a rule that is purely proof-theoretic according to Gauker, but it can be done in purely proof-theoretic terms by using just (typed or sorted) languages instead of “domains”. I will show, moreover, that the validity of UI is compatible with his concerns about context-variance. In section 5, I argue that, in terms of preservation of classicality, that proposal is just as good as another option which seems closer to classical logic only because it respects the most usual, textbook versions of UI.

Before I proceed to the main discussion, some disclaimers are in order. First, Gauker’s is by no means the only attack to the validity of UI. It is regarded as invalid in, for example, free logics.² However, as I will mention in section 3, their reasons differ. In this paper I am concerned only with Gauker’s attacks on UI; whether and how my approach can be used to face some of the challenges motivating free logics or other attempts to undermine UI is a theme for a separate work. Second, in the vicinity of Gauker’s attack on UI there are some topics important for his overall philosophy which I cannot address here, such as the existence of propositions or the nature of linguistic communication. I have tried to isolate as much as possible the asymmetry problem from those other considerations, so the reader would not find any statement on them. However, if I succeed in section 4, one can solve the asymmetry problem in a classical setting yet one that is not *prima facie* incompatible with Gauker’s views on those other matters. Finally, it goes without saying that I do not claim that the solution to the asymmetry problem sketched here can pass every test it might be put under by different context-sensitivity proposals, but merely that it gives a satisfactory answer to the problem on Gauker’s terms.

² For a general introduction to free logics, see Nolt (2006).

2. *Approaches to the asymmetry problem*

One can distinguish at least two broad approaches to the asymmetry problem, namely those solving it by *defending* classical logic and those which face it by *attacking* classical logic.³ Defenses typically deflate the problem, while attacks consider it genuine and look for further considerations pertaining to the asymmetry between the failure of UI and the validity of EG. Defenses can be either *conservative* or *radical* with respect to both logical orthodoxy, which one can safely take to be classical logic, and intuitions about the ordinary usage of sentences, or “ordinary linguistic intuitions” for short. I will say a bit more about this in a moment.

The conservative defense runs as follows:

- (CD) The premise in the application of UI in the example is true, but it is elliptical of a sentence whose logical form is, say, “All the students currently enrolled in the class are present” (“For all x , if x is a student in the logic class, then x is present”). But if the premise is really a conditional proposition universally quantified, then the conclusion does not follow. Since UI would remain valid as well as EG, there is no asymmetry problem.

There are two radical defenses:

- (RD1) The rule of UI presupposes a contextually given domain of discourse. Accordingly, although the rule is valid, the inference is fallacious because in the context of the premise, the name “Hypatia” does not name anyone. UI would remain valid as well as EG, so there would be no asymmetry problem.
- (RD2) Strictly speaking, the premise is false. Clearly, not everybody is present in my classroom. But if the premise is false, then the argument in question cannot be a counterexample to UI. Again, UI would remain valid as well as EG there would be no asymmetry problem.

³ The approaches I am about to present can all be found scattered through Gauker (1997). In Gauker (2003; ch. 7) they are better grouped but they are not dealt with in equal detail and subtlety as in the previous text. In my reconstruction, both in this section, which merely describes the approaches to the asymmetry problem, and the next one where they are critically examined, I have tried to combine the better assembling and the conciseness of the latter with the required subtleties of the former. The division I propose here is not the only possible one. The approaches can be divided, for example, between those defending ordinary intuitions about uses of quantified sentences and those which attack them. As I will show later, this initial division is important only for expository purposes because in any case the proposals will be ordered in the end with respect to their degree of departure from both logical orthodoxy and ordinary intuitions.

As in the case of defenses, attacks to classical logic also can be either conservative or radical, depending on their degree of departure from both logical orthodoxy and ordinary linguistic intuitions. One of those *conservative attacks* (CA1) is the same as (RD1). What makes it an attack, although of a conservative kind, is the same thing that makes it radical among defenses: It proposes rewriting the rule to a certain extent to make it more precise. A second conservative attack is Gauker's preferred option:

- (CA2) The counterexamples to UI are genuine, they should be accepted as well as the asymmetry, and an explanation of the latter is needed.

The radical attack goes as follows:

- (RA) The counterexamples to UI are genuine, but there is no asymmetry: EG is subject to counterexamples as well.

I have said that certain approaches to the problem are either conservative or radical with respect to their degree of closeness to both logical orthodoxy and ordinary linguistic intuitions. The guide is roughly as follows: Each logical principle and ordinary linguistic intuition comes with a *rationality index*, so when logical principles and ordinary linguistic intuitions stand in an irresolvable conflict when trying to solve a problem, their indexes must be compared to choose between either of them, if any. Provided that one has good reasons for thinking that the assignments of rationality indexes are largely correct, one aims at theories that preserve as much as possible the assigned rationality indexes. Typically, logical principles have rationality indexes greater than those of ordinary intuitions, but this is not necessary.⁴

Prima facie, (CD) is the most conservative option, at least from the point of view of logic. It keeps logic unaltered and it seems to score relatively well regarding ordinary intuitions, as it just requires an adjustment in the idea that some sentences that seem universal sentences *simpliciter* are really conditionals quantified universally. (RD1) seems slightly less conservative from the point of view of logic, given that it saves the ordinary intuition regarding universal sentences but requires at least rewriting the rule to make contexts explicit. If logic has a greater rationality index than ordinary linguistic intuitions, then (RD1) seems to score slightly lower than (CD).

Note that the radical defense (RD2) seems, strictly speaking, more radical than any of the attacks. (RD2) leaves logic unaltered, but implies a general-

⁴ The assignment of rationality indexes is to be done on a case-by-case basis; for a general methodology of comparison between rationality indexes and rational choice based on them once they are assigned, see Priest (2006).

ized failure in uttering universal sentences that are strictly speaking true. The failure is so massive that no matter how high the rationality index of logic is, one cannot simply keep it if it implies such a generalized failure. On the other hand, (CA2) respects the ordinary linguistic intuition that those true universal sentences are indeed universal sentences and that they are true, but is radical from the logical point of view since it declares the invalidity of UI and the asymmetry between UI and EG. Thus, given that (CA2) respects ordinary linguistic intuitions, unlike (RD2), and that the logical failure in (CA2) seems less dramatic as the generalized failure in ordinary practice involved in (RD2)⁵, (CA2) is less radical than (RD2). Likewise, the radical attack (RA), losing both UI and EG, seems less radical than the generalized failure implied by (RD2). This appealing to appearances might seem insufficient, but given that I will defend the logical orthodoxy without drawing on (RD2), the exact order between them makes little difference for my case.

So, *prima facie* we have the following options, where ‘A > B’ means that A is theoretically more conservative than B:

$$(CD) > (RD1) = (CA1) > (CA2) > (RA) > (RD2)$$

However, I will argue in section 4 that, in the pertinent logical sense, (RD1) is as logically conservative as (CD), and given that it respects the ordinary linguistic intuitions, it should be preferred over (CD). Moreover, in giving also a satisfactory account of the apparent asymmetry (rejecting it), while saving as much as possible of both logical orthodoxy and ordinary linguistic intuitions, it should be preferred over the other options as well.

3. *Gauker’s cases against (CD), (RD1), (RA) and (RD2)*

Before presenting my case for (RD1), it will be convenient to present Gauker’s arguments against the approaches he considers flawed. Gauker offers three objections against (CD). First, he says that it “is not a way of defending universal instantiation; it is just a way of explaining the counterexample” (Gauker 2003: 150). Since UI “is a formal rule of inference pertaining to sentences”, if the appeal to ellipsis means that a *sentence* of the form “Every x is F ” expresses a conditional *proposition* universally quantified but a sentence like “ a is F ” express a proposition of its very same form, then UI is wrong, because it fails

⁵ Classical logic is definable in Gauker’s proposal, as he himself has pointed out, so classical logic is not entirely left out. Classical logic would be the logic for maximal contexts, but Gauker’s arguments would aim to show that one does not always reason in maximal contexts and that a logic for reasoning in those other contexts is needed.

to capture what sentences might express and thus sanctions a relation between sentences that might not obtain.

The second objection to (CD) is that there is no principled way of knowing what sentences are elliptical of what, nor a principled (syntactical) way to fill in the ellipses in a unique way; for example, “Everybody is present” could be elliptical for both “Everybody enrolled in the class is present” and “Everybody who has been attending recently is present”. Such a way to fill in the ellipses should be therefore in the speakers’ minds, but they do not always have in mind a definite way of filling in the ellipses, even if they may say something definite when uttering apparently elliptical sentences (Gauker 1997: 192f). Then (CD), in imposing a logical form, would distort what speakers actually say.

Gauker’s third objection to (CD) is that it does not explain why there are no apparent counterexamples to EG, and in that sense the asymmetry persists (Gauker 2003: 153). If “Everybody is present” is elliptical for “Every student currently enrolled in the class is present”, there is nothing to be gained in saying that “Somebody is present” is elliptical for “Some student currently enrolled in the class is present”: Both would follow from “Hypatia is present” –the former directly and the latter after having made explicit the ellipsis for the premise too– if the teacher claims it after having taken attendance.

Against (RD1), Gauker offers two considerations. First, that it “reveals an error about the content of the rule of universal instantiation” because the rule does not mention any contextual constraint. It says that we can always replace the variables in question with *any* name in our language; see Gauker (1997: 186), (2003: 153).

As I have mentioned in connection with (CD), Gauker thinks of UI as a formal rule of inference pertaining to sentences. So a rule like “From $\forall xP$, infer $P(t/x)$ so long as t denotes an object in the domain of discourse and with all the usual restrictions” could seem to him a departure of such formality of the rule to introduce heavy model-theoretic notions like “denotation”.⁶ Additionally, even if “denotation” is re-phrased in more proof-theoretic terms, it still could be taken as a different rule, not the original UI, because Gauker thinks of UI as totally liberal about the substitutions.

Nonetheless, Gauker discusses whether any appeal to “standard model theory” could help to support (RD1). He says that the fact that standard model theory uses domains for interpreting languages would not do the required job, because “the presence of such a domain in a model-theoretic interpretation does not acknowledge any kind of context-relativity in the meaning of the quantifiers” (Gauker 2003: 152; see also 1997: 186).

⁶ A rule like that appears in Barwise and Etchemendy (1999: 319, 321).

On (RA), Gauker says that the purported counterexamples to EG fail. Let me introduce his view by presenting his differences with free logicians. Gauker and the free logicians share distrust on UI but, unlike free logicians, Gauker thinks EG is valid. It is due in part to the fact that Gauker's case does not rest in non-denoting terms but, commenting explicitly on the free logicians' takes on EG (see Gauker 1997: 194), he says that a more reasonable option to handle names of fictions like "Santa Claus" is saying that sentences containing them are never strictly speaking true, although in certain contexts one may "make as if they were true". From this perspective, there is nothing wrong with EG: It does not lead from truth to falsehood, or from something that one may be treating as true to something one should treat as false.

Simply put, for Gauker (1997: 191; 2003: 157) the counterexamples to EG fail because UI is "risky" in a way that EG is not. According to him, "Everybody is present" establishes a context which can be shifted by the very utterance of "*a* is present" if "*a*" belongs to a different context. But when "*a* is present" establishes a context, it seemingly cannot be changed by a mere utterance of "Somebody is present". In general, considering only sentences and their components, universal sentences may have certain relevant argumentative properties (like that of fixing a certain context, or being acceptable in a context) which an instance of it may lack, but every property a particular sentence holds is also held by the corresponding existential statement, unless elements alien to the sentences are introduced.

Against (RD2), Gauker offers also two considerations. First, he says that if the premise were false, then "we would hardly ever utter universally quantified sentences that were strictly speaking true, which is not very plausible" (Gauker 2003: 154). This is the reason I have given to consider it the most radical option and the least preferable, so if there is a solution less radical than this, it should be embraced.⁷ Second, Gauker says that it does not completely solve the asymmetry problem. Suppose that universally quantified sentences are never true, as (RD2) says, but merely "express a truth" or "are acceptable" in certain contexts. Saying that sometimes universally quantified sentences are not strictly speaking true, and that both UI and EG preserve (strict) truth, does not explain why EG preserves the properties *expressing (a truth in a context)* or *acceptability (in a context)* while UI does not (Gauker 1997: 187; 2003: 155f).

Having eliminated all the other options, the remaining is (CA2): The counterexamples to UI are genuine, they should be accepted as well as the asymmetry, and an explication of the latter is needed. Roughly, Gauker's proposal in his "context logic" is that quantification is always over (*inter alia*) named objects,

⁷ Gauker points out that such a view might be endorsed by Bach (2000).

and once named an object is relevant. So UI need not be valid, since a need not have been named, but EG must be valid; the asymmetry emerges quite naturally in his theory.

Nonetheless, in the following section I will show that Gauker's arguments against (RD1) are inconclusive and that, moreover, (RD1) is the best solution to the asymmetry problem. Concretely, Gauker has taken the "standard" or "more common" version of UI as if it were the only one and the most general one, which it is not.⁸ I will also present in more detail his context logic and show that it does not really invalidate UI in its most general form, which allows for context-relativity. Insofar as the ordering that I have provided between the approaches is correct, an argument establishing such an option less theoretically expensive than Gauker's is enough for my purposes; for all I know Gauker could be right in his arguments for ruling out the other approaches, but my proposal would not be affected if they are not.

4. *Arguing for (RD1)*

Gendler Szabó (1998: 1610) hinted informally at a solution to the asymmetry problem and to Gauker's arguments against UI:

One might say instead that the logical form of universally quantified sentences in English contains a hidden parameter whose value is fixed by the context in which the sentence is uttered. If we incorporate the appropriate context-sensitive clauses in our semantics of English, we can maintain the classical definition of validity.

Moreover, Gendler Szabó granted Gauker that although there are no real instances of UI failing to hold, there is an asymmetry between UI and EG, for UI is risky in a way EG is not, but that it is doubtful that "this is a problem for *logic*". I think Gendler Szabó is on the right track and I will try to spell out these ideas out more clearly.

Let me start with a language of (first-order) classical logic. I assume that the reader knows what such a language is like, but it is important for my purposes to state what its components are. A formal first-order language is a non-empty set of formulas recursively defined on a (non-empty, possibly infinite) fixed formal vocabulary whose building blocks are a (non-empty, possibly infinite) set of terms (constants, variables and functions), a (non-empty) set of sets of n -ary predicates for each natural number n , and a (non-empty, possibly infinite) set of n -ary connectives, which include the quantifiers.

⁸ Think for example that his case against model-theory helping with context-relativity only covers standard model theory with a unique domain and so on.

It is customary to think of such a language as *unique* and *homogenous*, that is, as being the only language in which derivations are made and that such language does not have further kinds, sorts or types. For example, the distinction between terms ends in the distinction between constants, variables and functions; there are no further kinds of variables, constants or functions.⁹

Thus, transformation rules operate on that language and only on that language. The usual rule UI is formulated for that language as follows:

$$\text{UI (in } L'): \frac{\text{For all } x \text{ (in } L), P \text{ (in } L)}{P(t/x) \text{ (in } L)}$$

where $L \subseteq L'$. That is, rules are applied in a language or for a kind of expressions at least as expressive as that in which the expressions are being formulated. More formally:

$$\text{UI}_L: \frac{(\forall x_{\in L} P)_{\in L}}{(P(t/x))_{\in L}}$$

But since the language is taken to be unique and homogeneous, $L' = L$ and it is not necessary to make it explicit:

$$\text{UI: } \frac{\forall x P}{P(t/x)}$$

That is, the usual rule.

But when neither the uniqueness of the language nor its homogeneity are taken for granted, languages or their further kinds, sorts or types must remain explicit in writing the rule:

$$\text{UI}_L: \frac{(\forall x_{\in L} P)_{\in L}}{(P(t/x))_{\in L}}$$

The same rule as before, but the subscripts are not eliminable because it might be that $L' \neq L$. The rule has not changed, as Gauker could reply, but what have changed are the assumptions about the language which stand in the way of giving its more usual, textbook formulation. Similar relativizations are required for EG, and there would be no logical reason to think that UI is more risky than EG, since a context-shift would amount to the same logical mistake in both cases.¹⁰

⁹ The distinction between “free” and “bound” variables is a description of the structure of certain formulas, not a distinction between kinds, sorts or types of variables, variables which deploy radically distinct logical roles: It is not that only some variables could be free and others bound, but they appear free or bound according to the structure of the formulas.

¹⁰ It is worth mentioning, although I will not dwell on this here, that this proposal is akin to those

Dropping whether the uniqueness or the homogeneity conditions does not directly lead us to “non-classical” logic. True, free logic is an example of dropping the homogeneity conditions, but there also is classical type theory and many-sorted classical logic.¹¹ Moreover, these restricted quantifiers are very common. In mathematical practice, for instance, sentences like “For all real numbers, so and so” are almost never symbolized as conditionals with antecedent “ x is a real number” universally quantified, but as a universal sentence under a typed or sorted language, as can be verified in many textbooks and, again, classical logic is almost always at play in mainstream mathematics.

So now I am in position to say what is wrong with the alleged counterexamples to UI. Gauker is wrong in making first a language-restriction in the premise and then supposing that, in spite of that restriction, one is still allowed to access or use a different, larger part of the language:

$$\text{UI}_L: \frac{(\forall x_{\in L} P)_{\in L}}{(P(t/x))_{\in L}}$$

Then, no wonder why UI “fails”: “Everybody is present” holds at the part of the language corresponding to the logic class context, L . But it is said that the x 's can be taken from a larger part of the language, L' , for example the name “Hypatia” of the context of the whole world, and that nonetheless the resulting substitution should be valid in the restricted part of the language corresponding to the logic class. But that is not what UI says. Universal sentences establish a context, as Gauker says, but in doing so they also establish the pertinent names that will count as *all* names; that the inference is carried out in a larger context does not matter, because although it operates in a certain context, the rule applies to the quantifier and it says what are the x 's that can be instantiated. More than providing counterexamples to UI, Gauker has fought the wrong rule one must have once the uniqueness or homogeneity assumptions are given up.

in philosophy of logic according to which what differs between logics needs not be the meaning of connectives, determined by their rules in some proposals, but something structural about the context of derivation in general. For this sort of proposals see Paoli (2003) and more recent discussion in Hjortland (2013).

¹¹ The literature on these topics is immense, so I just will mention some texts which I think are accessible for philosophers. For classical many-sorted logic, see Gallier (2003). For classical type theory, Andrews (2002), chapter 5; the seminal Henkin (1950) is still useful, and it nicely relates type theories and many-sorted logics. The first sections of Lambek and Scott (1986: part II) are also accessible and although they start with intuitionistic type theory, they mention what is needed to obtain the classical version. The rule UI_L (modulo notational variations) can be found in all of them. Notoriously, it remains unaltered when going from intuitionistic type theory to classical type theory in Lambek and Scott (1986), as its validity is independent of the validity of $p \vee \neg p$; this is important because in Gauker's context logic $p \vee \neg p$ fails too but, as I will show, UI_L holds.

A Gaukerian might retort that it is implausible to suppose languages come equipped with a type for every context, so the proposal presented here would not be really helpful. But this is a different, more general topic, namely the problem whether ordinary languages neatly possess all the required structure, which is neatly present in their idealized, formal counterparts. Moreover, this kind of assumption is not alien to Gauker's proposal: He too presupposes that his formal contexts, to be described below, have the resources to represent at least the essential features of real contexts. Given that the two assumptions are not very dissimilar but the rest of Gauker's proposal requires more changes in logical orthodoxy while mine does not require equally theoretically expensive changes neither in logical orthodoxy nor in our intuitions about ordinary language, I think the objection would not be telling.

I will end this section pointing out that UI_L is valid in Gauker's *context logic* in the following form. According to Gauker¹², a *context* (of assertibility and deniability) Γ consists of a set of names N_Γ , the (syntactic) *domain* of Γ , and a set of literals (atomic formulas or their negations) B_Γ such that not both p and $\neg p$ belong to B_Γ (although both can be absent for some p): This is the *base* of Γ . p is assertible in Γ iff and only if $p \in B_\Gamma$. Other assertibility and deniability conditions are determined recursively but I will not mention all of them here because my focus is just on the universal quantifier, whose assertibility conditions are as follows: $\forall x_{\in NT} P$ is *assertible* in Γ if and only if for every $t \in N_\Gamma$, $P(t/x)$ is assertible in Γ . Then

$$UI_\Gamma: \frac{(\forall x_{\in NT} P)_{\in \Gamma}}{(P(t/x))_{\in \Gamma}}$$

is valid in Gauker's sense (assertibility-preserving in a context), but this is just a form of UI_L , so the non-classicality of context logic does not come from the (supposed) invalidity of universal instantiation. The only change needed in Gauker's theory to get classical logic is making all contexts maximal, i.e., such that for any p , p or $\neg p$ belongs to B_Γ . Again, the rules do not change, only the assumptions about the contexts. Paraphrasing Gauker's own diagnosis on a certain Kaplanesque semantics for free logic (see Gauker 1997: 197), one could say that context logic is basically correct in content but wrong in form.

¹² For the details of his context logic, see Gauker (1997: 208ff). In what follows I present it rather succinctly, making just a few, unessential changes in the exposition.

5. (RD1) OR (CD)?

As I have said, (RD1) saves the intuition that common universal sentences are indeed universal sentences and also saves classical logic, with the exception that the uniqueness and homogeneity conditions on a language are not taken for granted. (RD1) is formally equivalent to saving the whole of classical logic, including the uniqueness and homogeneity conditions on a language at the expense of not saving certain appearances, that is, (CD). Every formula of the form $(\forall x_{\text{el}} P)_{\text{el}}$ can be transformed into a formula of the form $(\forall x(D \supset P))_{\text{el}}$, with $L \subseteq L'$. If L' is furthermore regarded as unique, then it can be written simply as $\forall x(D \supset P)$. The transformation for $(\exists x_{\text{el}} P)_{\text{el}}$ is $(\exists x(D \wedge P))_{\text{el}}$, with $L \subseteq L'$. If L' is moreover regarded as unique, then it can be written simply as $\exists x(D \wedge P)$. The process can of course be reverted and is left as an exercise to the reader.¹³

This intertranslability result implies that classical logic is presentation-independent, whether using a unique and homogenous language, a unique but typed, sorted language or several homogenous languages. All of them, together with certain additional common assumptions on, say, the number of truth-values and the functionality of interpretations, give rise to classical logic. However, arguments for one of the options and against the other can be put forward because the formal equivalence forgets the rationality indexes associated with each of the components of the proposal. For those associating a higher rationality index with saving the appearances, (RD1) is more appealing, whereas for those assigning a higher rationality to keeping most of the usual assumptions of classical logic, like Mion (2014)¹⁴, (CD) is better.

But what would be those “most usual assumptions of classical logic”? One should distinguish at least roughly between the necessary components of a logic and those aspects pertaining only to a presentation of it. From the point of view of keeping classical logic, (CD) is not more conservative *simpliciter* than (RD1), but only more conservative with respect to a *presentation* of classical logic. It can be argued that this presentation has a rationality index greater than those of the other presentations, but this is not entirely right. For example, an argument based on the parsimony of the standard presentation could go as follows. The intertranslability argument allows saying that “nothing essential that can be done with them [many sorts, types or languages] that cannot already be done without them.” (Enderton 2001: 297) For simplicity, when

¹³ For details, see Enderton (2001), chapter 4.

¹⁴ Actually, Mion (2014) defends that either (CD) or (RA) are the best solutions to the asymmetry problem, while accepting Gauker’s criticisms to (RD1). But if I am right, there is no need to even consider (RA).

one thing is enough, one should not accept more. But the most general formulation of classical logic, or of UI at any rate, does not require accepting more than one language, or more than the types or sorts associated with that unique language, but merely accepting that there is one while remaining neutral on whether there is exactly one. “There are exactly n things” is actually more loaded than “There are at least n things” –both are true if there are n things, but the former can be false in more cases than the latter–, so whenever the latter could be adopted in formulating a logical notion, it should be adopted. That the additional components of classical logic considered together make other languages or further types or sorts logically dispensable is after the fact; certainly an independent formulation of UI requires the most general formulation available and that is the one not assuming just one language or just the types or sorts usually associated with that unique language.

A unique and homogenous language may be common in philosophy and convenient for certain philosophical enterprises, but the other presentations are more common in computer science and the other home for classical logic together with philosophy, namely mathematics (with the probable exception of mathematical practice interested in monolithic foundations). It can also be said that parsimony regarding the number of languages or of further types or sorts certainly could be a virtue, but parsimony is not necessarily the decisive virtue for choosing a theory, especially if parsimony implies losing simplicity –as could be in the case of mathematical practice– or adequacy to the data–as in the case of ordinary uses of universal sentences. Therefore, if (RD1) keeps classical logic and reflects better the usage of universal quantifiers not only in ordinary language usage but also in many other cognitive enterprises, it should be preferred over (CD) for dealing with the asymmetry problem, even if the latter is parsimonious regarding the number of languages, or sorts, or types.

6. *Conclusions*

Christopher Gauker has presented some cases which seemingly undermine the validity of UI but nonetheless leave EG untouched, and has developed a very sophisticated theory to explain why this is so. He rejects several attempts to save the logical orthodoxy, especially those aiming at showing what would be wrong with the counterexamples. I have argued here that some of those proposals are better grounded than Gauker thinks and that ultimately they should be preferred over his since they explain satisfactorily the counterexamples without the revisions demanded by Gauker’s theory.

In particular, a proposal according to which (the terms which can be substi-

tuted for the variables bound by) quantifiers are restricted to languages –when there is no guarantee that there is a single language– or to additional sorts or types of expressions of a single language –when there is no guarantee that it is homogeneous– can explain the counterexamples while maintaining classical logic. Counterexamples arise from wrongly assuming that the original rule could sanction certain relations between languages (or sorts, or types) when it would not. Gauker’s objections to this proposal mistake a common formulation of the rule for a unique and most general one. I have argued that this proposal scores better in more criteria for acceptability than the others, so it should be preferred as a solution for the asymmetry problem.

Among the open problems left for further work is whether and how my approach can be used to face some of the challenges motivating free logics or other attempts to undermine UI. I have noted in passim that my proposal is akin to those that identify the differences between logics not in the rules for operating connectives but in more “structural” aspects of the contexts of derivability. Exploring this idea along the lines sketched here is also left for further work.

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