

# Wittgenstein: Necessity, Imagination, and Metaphilosophy

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*Abstract:* *Prima facie*, there seems to be a tension between Wittgenstein's Kantianism in the philosophy of mathematics (especially the view according to which a proposition  $p$  is logically necessary if we are not able to conceive of *not-p*) and Wittgenstein's later use of imagination in philosophy (especially his building *logically alien* cases, in which certain imaginary people *reject* what we take to be a logically necessary proposition). This paper investigates the tension just sketched by assessing and criticizing three answers to the question "With what purposes does Wittgenstein build logically alien imaginary cases?". One of the main reasons why the paper rejects such answers is that they are not compatible with Wittgenstein's later meta-philosophy. The paper also sketches an alternative account, on which the tension between Kantianism and philosophical imagination in Wittgenstein's later philosophy of mathematics is only apparent, and argues that, thus interpreted, Wittgenstein's use of imagination, far from being in contrast with Wittgenstein's meta-philosophical ideas, somewhat derives from them.

## 1. *Kantianism v. philosophical imagination*

As is well known, in 1931 Wittgenstein made a list of ten people who had influenced him, but he did not include Kant in the list. In fact, he once suggested that from Kant he could get only a vague understanding and occasional insights (von Wright 1955: 543). However, Wittgenstein had the normal educated man's acquaintance with Kant (McGuinness 1988: 252-253), and he assimilated Kant's critical philosophy through Schopenhauer, Hertz, Boltzmann, Frege, and even Russell (who were all on the list). Moreover, in 1916, he read the *Critique of Pure Reason* together with his friend Ludwig Hänsel. It is thus not surprising that several scholars have detected methodological and substantial similarities between Kant and Wittgenstein (Glock 1997).

In this paper, I shall not make a detailed comparison between Kant and Wittgenstein, but I shall confine myself to a somewhat general anal-

ogy between them, namely, their shared view of the modal status of necessary propositions. In particular, it is widely accepted that Wittgenstein's later views on logical necessity, that is, the necessity of mathematical propositions (broadly conceived as including logical laws, arithmetic equations, and geometric axioms and theorems) belong to a broadly *Kantian* tradition in the philosophy of mathematics, according to which such necessity does not inhere in things (belonging either to our world or to a Platonic realm), but is a by-product of our conceptual scheme: the necessary is what we could not conceive as being otherwise (for this formulation see Marconi 2010: 139; but this very general point is shared not only by Williams 1974 and Lear 1984, but also by different interpretations such as, for example, Putnam 1994, Conant 1991, and Forster 2004).<sup>1</sup> In other words, Wittgenstein accepts the following, broadly Kantian, view of logical necessity: a proposition  $p$  is logically necessary if we cannot conceive of *not-p* (1937-1944/1983: IV, § 29).

In Wittgenstein's view, logical laws and mathematical propositions are (disguised) *grammatical* rules and therefore have "the hardness of the logical must" (1953: § 437). The necessity of such rules goes hand in hand with the inconceivability of what would correspond to the opposite, but not because the "powers of imagination are unequal to the task" (§ 251). Wittgenstein warns us not to confuse a grammatical proposition ("Every rod has a length") with an empirical one ("This table has the same length as the one over there"). Only in the latter case do we understand what it means to have a picture of the opposite, whereas in the former case "the picture attaching to the grammatical proposition could only show what is called 'the length of a rod'" (§ 251).

However, there is as much evidence that, in striking contrast with the *Tractatus*,<sup>2</sup> the frequent use of thought experiments is one of the hallmarks of Wittgenstein's later writings. On the one hand, there are imaginary cases that are quite similar to real or familiar cases – call them "Logically Similar Cases" (hereinafter LSCs). For example, it is easy to conceive that people "should never speak an audible language, but

<sup>1</sup> In the first *Critique* Kant regarded mathematical propositions as paradigmatic cases of necessary propositions, that is, propositions that could not be otherwise, and argued that all a priori knowledge is of necessary propositions (1781-87/1999: A xv), and that all necessary propositions are knowable a priori (B3).

<sup>2</sup> One of the rare places in the *Tractatus* where Wittgenstein explicitly appeals to imagination for philosophical purposes is proposition 6.1233 ("It is possible to imagine a world in which the axiom of reducibility is not valid"). However, he provides no details of such an imaginary scenario (Peach 2004).

should still say things to themselves in the imagination” (1953: § 344); a language consisting only of orders and reports in battle, or a language consisting only of questions and expressions for answering yes and no (§ 19); a society of people who have a word, say, ‘P’, to which they respond in two different ways when one of their group complains of ‘P’, depending on whether the complaining person has or hasn’t suffered some visible injury (1967: § 380); a language in which “there is not a form for questions, or commands, but they are expressed in the form of statements” (1937-1944/1983: app. III, § 1); “a game very akin to chess, consisting in making chess moves, but without there being any winning and losing in it; or with different conditions of winning” (app. III, § 2); and so on and so forth.

On the other hand, especially in Wittgenstein’s later reflections on the philosophy of mathematics, there are many more bizarre imaginary cases (call them “Logically Alien Cases”, hereinafter LACs), in which certain imaginary people – call them “logical aliens” – *reject* (in a sense that I would like to clarify in this paper) what we take to be a logically necessary proposition (or, at least, they behave in a way that implies such a rejection). For example, there might be people with a more primitive logic, in which only for certain sentences is there anything corresponding to our negation (1937-1944/1983: app. I, § 8; 1953: § 554; 1939/1976: 179); or a formal logic that ceases to acknowledge the law of contradiction (1937-1944/1983: IV, § 59). Or else: people whose numbers only go up to 5 (1953: § 555; 1937-1944/1983: III, 84); a situation in which it emerges that we have always gone wrong up to now in multiplying  $12 \times 12$  (1937-1944/1983: I, § 135); people who measure with elastic rulers (I, § 5); a society acting as follows: they pile timber in heaps of arbitrary, varying height and then sell it at a price proportionate to the area covered by the piles; they justify their behavior with the words: “Of course, if you buy more timber, you must pay more” (I, § 149); someone bewitched so that he calculates  $4 \times 3 + 2 = 10$  (I, § 137); or people who “go through one of our calculation to-day and [are] satisfied with the conclusions, but to-morrow want to draw quite different conclusions, and other ones again on another day”, so that when they “make *this* transition one time, the next time, ‘just for that reason’”, they “make a different one, and therefore (say) the next time the first one again” (I, § 155); a man who checkmates with a king and a knight (1939/1976: 148)<sup>3</sup>; and so on and so forth.

<sup>3</sup> Here and in what follows, I have modified Wittgenstein’s original example (which concerned mating with a *king and two pawns*), for it is possible and, in fact, quite easy to check-

A quite widespread account of Wittgenstein's use of imagination in philosophy draws a sharp divide between the two kinds of thought experiments. On the one hand, Wittgenstein's LSCs are conceptually similar to familiar or real cases. But we are normally able to conceive and understand concepts similar to ours. Therefore, Wittgenstein's LSCs are conceivable and intelligible (Canfield 1975). Since Wittgenstein usually takes it for granted that conceivability entails possibility, it follows that, in Wittgenstein's view, LSCs are possible. From this conclusion it is quite natural to draw the following, further consequence: the very purpose of Wittgenstein's LSCs is, as it were, *to highlight (or reveal) possibilities*.<sup>4</sup>

On the other hand, in Wittgenstein's LACs, certain imaginary aliens *reject* what we take to be a logically necessary proposition, say  $p$ . In other words, such aliens *deny*  $p$ , accept or assent to *not-p*, or even assert *not-p*.<sup>5</sup> But since Wittgenstein accepts the broadly Kantian view, according to which  $p$  is logically necessary if we cannot conceive of *not-p*, it follows that, according to Wittgenstein, the circumstances described by LACs are ultimately inconceivable (and unintelligible). But then it is natural to ask: *Why, for what purposes does Wittgenstein build LACs, that is, ultimately inconceivable and unintelligible imaginary cases?* For example, one can imagine and conceive that a logical alien behaves in a certain way or utters certain sentences, for example, he makes certain moves on the chessboard using a king and a knight and suddenly cries "Checkmate!", or he writes " $2+2=5$ " on a piece of paper. But if it is necessary that  $2+2=4$ , if it is impossible to checkmate with a king and a knight, then it is strictly speaking unintelligible and inconceivable for someone to calculate  $2+2=5$  correctly or checkmate with a king and a knight. So why should one describe the imaginary cases of someone calculating  $2+2=5$  correctly or checkmating with a king and a knight?

In what follows, I shall consider and assess three possible answers to the question: "For what purposes does Wittgenstein build such LACs?" (sections II, III and IV). Each answer has drawbacks and is, in particular, not readily compatible with Wittgenstein's explicit metaphilosophy. I shall then sketch an alternative account of Wittgenstein's thought experi-

mate with a king and two pawns: it is likely that Wittgenstein was not thinking of the possibility of *pawn promotion* (converting a pawn, say, into a queen). Thanks to an anonymous referee for suggesting this.

<sup>4</sup> As is well known, the issue is hotly disputed (see, for example, Gendler and Hawthorne 2002). Accordingly, this paper neither provides a comprehensive discussion of it nor (perhaps more importantly) takes a Wittgensteinian stance on the question.

<sup>5</sup> Here I take assent to be a *mental* act, and assertion to be a (sincere) *speech* act.

ments, in which the tension between Wittgenstein's use of imagination and Wittgenstein's broadly Kantian conception of necessity is only apparent (section V). I will also argue that Wittgenstein's use of imagination, far from being in contrast with Wittgenstein's metaphilosophical ideas, to some extent derives from them (section VI).<sup>6</sup>

## 2. Frege and the awareness of illusion

In Kant's view, the rules of logic are the preconditions of the possibility of judgment (1781-87/1999: A52/B76, 109-10): not just finite human judgment, but rather judgment *as such*. The *Jäsche Logic* provides us with a quite perspicuous summary of Kant's conception of *general* logic (conceived of as "the science of the necessary laws of the understanding and reason in general, or – which is the same – of the mere form of thinking") (1800/1992: §§ 13-16): logic is formal, that is, it is concerned with the form of coherent thought; it abstracts from all objects and, generally speaking, from semantic content; it exhibits "the conditions under which alone understanding can and shall agree with itself"; in logic, "the question is not one of *contingent* but *necessary* rules, not how we think but how we ought to think"; logic must be sharply distinguished from psychology and natural sciences (as well as from special and transcendental logics); but it does not furnish metaphysical knowledge either (in fact, it does not provide substantial knowledge of any sort); logic has a special status, for it is a *canon*, that is, it is *constitutive* of reason and understanding. To put it in a slogan: *illogical thinking is not thinking at all*.

Nearly eighty years after Kant's death, Frege adopted a multi-faceted attitude toward Kant's conception of logic. On the one hand, he departed from it in several respects. The best-known difference lies in Frege's logicist thesis, according to which arithmetic is reducible to logic. The most dramatic difference lies in Frege's revolutionary use of quantifiers. The most philosophically profound difference derives from Frege's idea that logical truths are substantive rather than merely formal (MacFarlane 2002). On the other hand, however, Frege inherited the Kantian conception of logic in other respects. Not only did he agree with Kant that logical truths are maximally general, but he also inherited the Kantian idea that

<sup>6</sup> Though some of the issues discussed in the paper might be relevant to some contemporary debates in the philosophy of mathematics (see Horsten 2012), the paper's purpose is to clarify the relation between Kantianism and the use of imagination *in Wittgenstein's later philosophy*.

laws of logic are constitutive of thought and rationality. Let me elaborate on the latter point. Frege refers to the laws of logic as “the laws of truth” and “the laws of thought” and, following Kant, he thinks that the confusion of the logical with the psychological is a “widespread philosophical disease” (1884/1984: 368-9). Accordingly, Frege (1893-1903/1982) builds a thought experiment concerning the possibility of discovering a “logical alien”. He writes: “What if beings were ... found whose laws of thought flatly contradicted ours and therefore frequently led to contrary results even in practice? The psychological logician could only acknowledge the fact and say simply: those laws hold for them, these laws hold for us. I should say: we have here a hitherto unknown type of madness” (14).

One way of interpreting Frege’s thought experiment is to say that he aimed to show that such an imaginary situation, which the psychologistic logician finds intelligible, is ultimately unintelligible and inconceivable. Frege’s key move is to ask: Who is *right* (the logical aliens or ourselves)? Whose laws are *correct* (theirs or ours)? The point is that such questions cannot be psychological questions. Rather, they presuppose the possibility of assessing the scenario in normative rather than in merely descriptive terms. In fact, the very notions of agreement and disagreement, logical and illogical, and even thought are normative rather than descriptive. Psychologism gives us the illusion that we can describe logically alien thought, that is, illogical thinking (a thought that *disagrees* with our laws of logic). The psychologistic logician, however, is in no position to tell us anything about the *thought* of the logical aliens. For without logical, rather than merely psychological resources, one cannot describe someone’s thought but only his outward behavior. Under this interpretation, Frege’s anti-psychologistic thought experiment is ultimately based on the Kantian thesis, according to which illogical thinking is no thinking at all (Conant 1991, 137).

However, Frege also held another view, according to which the laws of logic *do* have a positive subject-matter: they are the most general laws of nature, which state absolutely general substantial truths. It is controversial whether Frege’s Platonism is in tension with his partly Kantian view of logic. In this paper, I do not wish to take part in such a controversy (MacFarlane 2002, Reck 2005). Suffice it to say how Wittgenstein, in the *Tractatus*, resolved Frege’s alleged instability by returning to Kant. In Wittgenstein’s view, the propositions of logic are tautologies that say nothing: they are formal, empty, a priori. Logic is not a doctrine: in fact, “Theories of logic which make a proposition of logic appear substantial are always false” (1921-1922/1974: 6.111). The main misunderstanding

(common to Frege, Russell and Psychologism) lies in thinking that, since logic is *true*, it must have a positive subject-matter. Thus, in a sense, Wittgenstein can be seen here as returning to Kant's thought that logic cannot deliver knowledge and that illogical thinking is not thinking at all ("Thought can never be of anything illogical, since, if it were, we should have to think illogically", 1921-1922/1974: 3.03).

According to James Conant, what one might call the Kant-Frege-*Tractatus* line of thinking in the philosophy of logic is shared by the later Wittgenstein (and, more recently, by Putnam 1994). Conant suggests not only that Wittgenstein thinks that the laws of logic are constitutive of thought and reasoning, and therefore cannot be abandoned except at the cost of abandoning, at the same time, thinking and reasoning (Wittgenstein 1937-1944/1983: I, 131, 132, 133, 134, 156, 163; IV, 29; VII, 38); but that Wittgenstein (1953) formulates the task of philosophy as follows: "The great difficulty here is to represent the matter as if there were something one *couldn't* do" (§ 374). Hence, the moral that Conant draws from the tale is threefold (1991: 157). Firstly, the later Wittgenstein fully belongs to the Kant-Frege-*Tractatus* line of thinking in the philosophy of logic, for he accepts the idea that illogical thinking is not thinking at all. Secondly, Wittgenstein's main goal in (his later) philosophy is the same as Frege's main goal in his anti-psychologistic thought experiment: to make one *aware* that one is subject to the illusion of thought. Thirdly, according to Wittgenstein (as well as according to Frege in the thought experiment discussed above), this goal should be pursued by means of philosophical *elucidation* (rather than of philosophical, substantial theory).

Such theses invite one to provide the following Conant-inspired answer to the question: "For what purposes does Wittgenstein build ultimately unintelligible LACs?": *in order to emphasize the illusion of understanding and conceiving* (as Frege did in his anti-psychologistic thought experiment). He constructs an imaginary case, in which, for example, a man checkmates with a king and a knight. *Prima facie*, such a situation looks as if it is conceivable and intelligible, as much as an LSC. In fact, this LAC is ultimately inconceivable and unintelligible: we cannot make sense of it. Its purpose is to make one recognize the illusion of intelligibility (that is, the illusion that such a case is intelligible) *as an illusion*.

Let me briefly comment on this. On the one hand, it is true that Wittgenstein recognizes that there is something that we might call "the illusion of understanding". For example, he writes: "Don't I also sometimes imagine myself to understand a word (as I may imagine I understand a kind of calculation) and then realize that I did not understand it?"

(“I thought I knew what ‘relative’ and ‘absolute’ motion meant, but I see that I don’t know.”)” (1953: § 138). On the other hand, however, Wittgenstein explicitly tells us that if we try to think the impossible (for example, if we look at our lamp and say: “This lamp is different from itself”), we do not succeed and can even get ourselves into a thinking-cramp (1937-1944/1983: I, § 132). It is worth noting, here, that he does not tell us that, by trying to think the impossible, we *become aware* that such an exercise of imagination is nothing but a mental cramp.

Moreover, there was no need for Wittgenstein to use imagination in philosophy in order to give up the idea of putting forward substantial philosophical theories: an insubstantial, *descriptive* conception of philosophy would have been enough (more on this in section VI below). As a matter of fact, however, he built many different and bizarre imaginary cases. It is highly implausible that he did so with the very purpose of doing what Frege did with *just one* thought experiment (let alone what the *Tractatus* did using *no* thought experiments at all), that is, exhibiting nonsense in order to make us aware of the illusory conceivability of a certain imaginary scenario, based on the Kantian assumption that illogical thinking is not thinking at all. Wittgenstein once complained that “Frege says in the preface to the *Grundgesetze der Arithmetik*: ‘here we have a hitherto unknown kind of insanity’ – but he never said what this ‘insanity’ would really be like” (1937-1944/1983: I, § 152). Given this remark, it is natural to wonder why Wittgenstein should criticize Frege for not providing examples and descriptions of logical madness, if – as Conant seems to suggest – such examples and descriptions do not make a difference, for they do not allow one to say or to show something *more than* or something *different from* what Frege said and showed in the anti-psychologistic thought experiment outlined above.

### 3. *Back to Descartes?*

In this section I shall take into account and comment on a different view, according to which Wittgenstein’s purpose (in building ultimately inconceivable LACs) is *to discover inconceivable possibilities, in a Cartesian mold*. Let me first explicate why and in what sense Descartes is involved here (see also Conant 1991).

As I have said above, Kant conceived of the laws of logic as the pre-conditions of the possibility of thought: not just *finite* human thought but, rather, thought *as such*. This qualification shows that one of Kant’s



main targets was Descartes. Descartes thought that Aquinas' and Suarez's idea, according to which "God is called omnipotent because He can do ... everything that does not imply a contradiction" (Aquinas 1265-1274/1947: Q. 25, art. 3) erred into blasphemy. According to Descartes, God could have made contradictories true together: God can bring about things which our minds are incapable of comprehending, and this is the only way to acknowledge His omnipotence. *From a Divine point of view*, logical laws are only *contingently* necessary. In fact, the necessity of logical laws depends on the fact that our minds are constituted in a certain way. But that our minds are so constituted is a contingent fact about our nature. Descartes refuses to say that God cannot make a mountain without a valley, or bring it about that 1 and 2 do not make 3. Rather, he says that God has given us a mind such that we cannot conceive a mountain without a valley, or a sum of 1 and 2 that is not 3 (1648/1984-91, vol. 3: 359) (such things involve a contradiction in Descartes' conception). But in his view, although it is unintelligible to us how God could have brought it about from eternity that, say, twice four makes eight, we must admit that it would have been easy for God to ordain certain things in such a way that we cannot understand the possibility of their being otherwise than they are (vol. 2: 294). Descartes alludes to the *queer* possibility of a radically different world, which is possible for God even though it is not intelligible for us (see also Wittgenstein 1939/1976: 147, where Wittgenstein apparently makes a similar move; but more on this below). According to Descartes, the possibility of such a world is not something we can *comprehend* (that is, we are not able to conceive such an imaginary world), but is something we can *apprehend* (that is, we can imagine that such an inconceivable world could be) (Geach 1977: 10).

Now, under the interpretation I discuss in this section, Wittgenstein describes in his LACs imaginary circumstances in which imaginary people reject a logically necessary proposition  $p$ , that is (in classical logic), they accept *not-p*. But even though, in continuity with Kant, he regards *not-p* as ultimately inconceivable and unintelligible, he thinks that *not-p* is, in a sense, *possible* (Stroud 1965 and Coliva 2010). This notion of possibility is metaphysical and anti-Kantian, for it implies that necessity is not a by-product of our ability to conceive and understand: in fact, it refers to the possibility of something inconceivable.

For example, Broyles (1974) succinctly tells us that some "of the many strands in the weave of Wittgenstein ... seem to deserve to be called metaphysical, in the best sense of that term" (296). More straightfor-

wardly, Coliva recently argued that Wittgenstein's imagining radically different communities leaves us with the "purely metaphysical possibility that if certain 'facts of nature' (PI II, xii) had been totally different, or if 'something really unheard of' (OC, § 513) were to happen, there could be creatures who ... don't reason as we do, for whom  $2+2$  isn't equal to 4, ... etc. Hence, all that remains to it is the metaphysical possibility that all this might happen, or might have been the case. Still, it is a possibility that we can't really conceive of in detail, given the kind of creatures we are and the fact that our concepts are what they are also because of some very general facts about us and about nature" (2010, 22).

As far as I can see, if these interpretations were right, Wittgenstein's ideas would resemble Descartes' view according to which, for example, God could have made contradictories true together, or could make it false that  $2 \times 4 = 8$ , so that, from the Divine point of view, even logical laws and arithmetical theorems are only contingently necessary (Alanen 1991); the possibility of God's bringing about from eternity that it was not true that, say,  $2 \times 4 = 8$  is something we can apprehend but not comprehend.

On the one hand, it is true that Wittgenstein's voice sometimes takes on such Cartesian tones. For example, he writes: "Imagine the following queer possibility: we have always gone wrong up to now in multiplying  $12 \times 12$ .<sup>7</sup> True, it is unintelligible how this can have happened, but it has happened" (1937-1944/1983: I, § 135). This seems to suggest there is the *possibility* of ways of thinking, counting, inferring, calculating, and so forth, alternative to ours, even though such alternatives are *unintelligible* and, strictly speaking, *inconceivable*. He also wonders whether it is impossible for someone to have always gone wrong in his calculation (even an elementary one such as  $2+2$ ), for a devil deceives one, so that one keeps on overlooking something, however often one goes over the sum step by step (I, § 136).

On the other hand, however, there are exegetical grounds for not attributing such a metaphysical, Cartesian view to Wittgenstein. First, if Wittgenstein were adopting such a notion of ultimately inconceivable, metaphysical possibility, that would put him *outside* the broadly Kantian tradition in the philosophy of mathematics, because he would cease to regard possibility and necessity as byproducts of our ability to conceive and think (whereas, as I said above, many people accept that

<sup>7</sup> I interpret this as meaning that we have always gone wrong up to now in calculating  $12 \times 12 = 144$ .

Wittgenstein takes a broadly Kantian perspective on logical necessity).<sup>8</sup>

Secondly, even though Wittgenstein usually takes it for granted that conceivability entails possibility, it is probable that he sometimes imagines something without explicitly wondering whether the imagined circumstance is, *strictly speaking*, ultimately conceivable and possible or rather inconceivable and impossible. Moreover, at other times he explicitly calls “conceivable” what a Kantian would regard as a logical impossibility. For example, he once reminded his students that if a man has a knight and a king, we give up playing, for there is nothing we will ever call “mating with one knight”.<sup>9</sup> However, Wittgenstein argued, a man may not give up trying. He may be convinced that there are still possibilities that we have not taken into account. In such a case, “is it inconceivable that he should one day do something which he and everyone else would call mating?” Wittgenstein answered: “No, not at all. He might do something which we now should call ‘not playing the game’ but of which people then would say ‘why yes, that’s all right’. – It seems to me immensely unlikely and I’m not going to gamble on it, but it’s conceivable” (1939/1976: 148). Here Wittgenstein does not try to *apprehend* by imagination the possibility of something inconceivable; rather, he seems to suggest that, however queer and unlikely, such a scenario is, after all, *conceivable*.

Thirdly, the thesis according to which Wittgenstein is attempting to apprehend certain inconceivable, metaphysical, Cartesian possibilities (or Kantian logical impossibilities) is difficult to reconcile with some aspects of Wittgenstein’s explicit metaphilosophy,<sup>10</sup> especially with his idea that one of the main goals in philosophy is to throw light on how our language works, in order to get *conceptual clarity* (Glock 2002). How could one get conceptual clarity by discovering or apprehending, by means of imagination, one or more metaphysically possible but *conceptually unclear* or even *inconceivable* scenarios?

<sup>8</sup> This remark, however, might sound question-begging, for one could also say that *there is* a tension between, so to speak, Kantianism and Cartesianism *within Wittgenstein’s views of logical necessity*.

<sup>9</sup> See footnote 3 above.

<sup>10</sup> For a comparison between Wittgenstein’s later philosophical method and his explicit metaphilosophical claims, see Williams (2010).

#### 4. *Making the inconceivable conceivable*

A third possible answer to the question: “For what purposes does Wittgenstein build LACs?” is based on Putnam’s interpretation of Wittgenstein’s views on logical necessity (Putnam 1992, 1994). According to Putnam, as the history of mathematics and natural sciences teaches us, there are cases in which we considered a certain statement, say  $p$ , as necessary, and we later discovered (or acknowledged) that  $p$  is, after all, contingent. In such cases, up to a certain time *not- $p$*  was (regarded as) inconceivable and unintelligible (hence, allegedly impossible), but it later became conceivable and intelligible (hence, surprisingly possible). Putnam’s favorite examples are non-Euclidean geometry and Einsteinian physics. Prior to Lobachevski, Riemann, and others no one knew how to disconfirm Euclidean geometry, or even if anything *could* disconfirm it: for example, the statement: “A plane triangle may have two right angles as base angles” was (considered as) inconceivable and unintelligible. However, as Putnam puts it, it was *true*. Learning Riemannian geometry enables us to give sense to those words. (This does not mean, of course, that we are stipulating a new meaning). Prior to Riemann, that triangle was (considered as) inconceivable. Therefore, it was regarded as impossible. After Riemann, we understand in what sense such a triangle is possible. By a similar token, prior to Einstein, the statement “Given two simultaneous events A and B, A happened earlier than B” was (regarded as) inconceivable and unintelligible. Einstein did not arbitrarily change the meaning of one or more words occurring in that statement. Rather, he gave them a sense. It is worth noting, here, that, as is well known, the use of imagination was fundamental for Einstein’s making sense of such a statement. It is natural to think, for example, of the very fast train thought experiment in Einstein’s 1917 *Popular Exposition of Relativity* (1917/1961).

When Putnam takes into account Wittgenstein’s question: “Are our laws of inference eternal and immutable?” (Wittgenstein 1937-1944/1983: I, § 155), his main points are the following. Firstly, non-Euclidean geometry and Einsteinian theory provide grounds for answering a qualified “No, at least in certain cases” to that question. Secondly, this is not to say that there is a metaphysical guarantee that something that will strike us as completely analogous to what happened in the case of geometry or physics will ever happen in the field of arithmetic or even of logic. (For example, the question: “Could one disconfirm the principle of non-contradiction?” does not have a clear sense yet). Thirdly, the first two points just stated express Wittgenstein’s own view about this issue.

This view of logical necessity (which Putnam attributes to Wittgenstein and shares with him) belongs to the Kantian tradition, as broadly conceived (the necessary is what we could not conceive as being otherwise), but Putnam brings it up to date in the light of non-Euclidean geometry and Einsteinian relativity. As far as I can see, Putnam comes quite close to Wittgenstein's later view of logical necessity, but a proper assessment of this point goes beyond the purposes of this paper. Here I am more interested in the following Putnam-inspired answer to the question: "For what purposes does Wittgenstein build LACs?": Wittgenstein builds LACs *in order to show us how one could make conceivable something that was, up to then, inconceivable, on the model of non-Euclidean geometry and Einstein's thought experiments.*

On the one hand, at first glance, the idea expressed by the Putnam-inspired answer is very attractive. First, in Wittgenstein's circle in Vienna, as well as in Cambridge, many people were discussing non-Euclidean geometry (as well as relativity theory), particularly friends of Wittgenstein's, such as Schlick and Russell (Penco 2010: 369). Moreover, both in Waismann's reports of the conversations at Schlick's house and in Wittgenstein's *Remarks on the Foundations of Mathematics* there are references to Riemannian geometry and to Einstein's point of view on geometry (1929-1932/1967: 38, 162; 1937-1944/1983: I, app. III, 7). In 1937, Wittgenstein wrote that Einstein's main contribution ("what Einstein taught the world") was the idea that the method of measuring time belongs to the *grammar* of time-expression (2000: MS 119; my emphasis); about five years before he had written that: "What Mach calls 'thought experiment' is of course not an experiment. At bottom it is a *grammatical* investigation" (2005: 441; my emphasis). Thus, one might conclude that, according to Wittgenstein (both "intermediate" and "late"), the similarity between Einstein's work and his own was that they were both conducting grammatical investigations by means of thought experiments (Penco 2010: 369).

Secondly, Wittgenstein famously suggested that if one believes that the concepts we use are absolutely the correct ones, so that someone holding different concepts would not realize something that we realize, then the philosopher should let him imagine certain general facts of nature radically different from the actual ones, and "the formation of concepts different from the usual ones will become intelligible to him" (1953: II, 230). Here the notion of facts of nature (which, as is well known, is connected with the concept of a *form of life*) should be taken in a broad sense, as including not only biological facts, but also more complex anthropological and symbolic ones. The following are examples of such

thought experiments: people learn sums by counting beans or apples or other things that appear and disappear by themselves (1937-1944/1983: I, 37, 137); things are measured with elastic rulers rather than with rigid ones (I, 5); a proof is engraved in rock but alters in appearance (III, 42); someone who sees, for example, the group I I I I as the group I I I I I with the two middle strokes fused, and therefore counts the middle stroke twice (I, § 169): imagining such worlds – one could say – allows us to understand or make intelligible ways of calculating, measuring and inferring etc. that we would find unintelligible otherwise.

Thirdly, when Wittgenstein (1937-1944/1983) comments on a puzzle consisting in making a particular figure, e.g. a rectangle, out of given pieces, he writes: “Can’t we say: the figure which shews you the solution removes blindness, or again changes your geometry? It as it were shews you a new dimension of space. (As if a fly were shewn the way out of the fly-bottle)” (I, 44). Quite interestingly, in the *Investigations* he uses the same metaphor to talk about *philosophy* (rather than *geometry*): “What is your aim in philosophy? – To shew the fly the way out” (1953: § 309). In both cases, the metaphor of being shown the way out points to what Wittgenstein regarded as a very important intellectual activity, namely, the attempt to avoid, “cure,” and overcome our “natural” blindness and/or our “instinctive” temptation (to think in certain misleading ways) (Janik 2011).

On the other hand, however, there are grounds for rejecting the Putnam-inspired interpretation just outlined. First, even if we grant, for the sake of argument, that some of the imaginary cases that I have just briefly described (especially those in which the things of the world – beans, apples, rocks, and so on and so forth – “behave” in non-ordinary, bizarre ways) make intelligible something up to then unintelligible, there is still a large number of cases that, at least *prima facie*, do not fit this pattern. Moreover, there are grounds for thinking that even those imaginary cases do not really work like that.

Wittgenstein does not tell us that such imaginary cases are meant to make intelligible *concepts* different from the usual ones; rather, he tells us that such imaginary cases are meant to make intelligible *the formation* of concepts different from the usual ones (1953: II, 230). The difference is telling. Such imaginary cases are not meant to provide us with *reasons* to understand why, say,  $2+2$  might be equal to 3: in fact, nowadays, even after being acquainted with Wittgenstein’s thought experiments, we still have *no* grounds at all for thinking that (it is possible that)  $2+2=3$ . Rather, such imaginary cases are meant to help us understand why some-

one could come to believe that, if he takes two beans plus two beans, he gets three beans: a possible *cause* of that belief is, for example, that in his world, as *we* would say, beans disappear without one noticing.

Secondly, the reference to the Einsteinian model is promising, though, as Penco (2010) has shown, one cannot do more than conjecture about the relation between Einstein and Wittgenstein, for there is not much material to work on; whereas the reference to non-Euclidean geometry is even more controversial, for it cannot be taken for granted that *imagination* plays (or played) a fundamental role in the acceptance of such alternative geometry.

Thirdly, according to Wittgenstein, the philosopher's "task is not to discover calculi, but to describe the *present* situation" (1937-1944/1983: III, 81), in order to make it possible for us to get a clear view of the present state of mathematics, without deducing or explaining anything (1953, § 125). In this sense, for example, the philosophy of mathematics *before* Riemann is and should be different from the philosophy of mathematics *after* Riemann. Hence, the philosopher should pay attention to the conceptual development of mathematics, in order to describe it appropriately. According to Wittgenstein, however, it is up to the mathematician, not the philosopher, to *create essences* (or necessities) (1937-1944/1983: I, § 32) and to open new possibilities (new roads) (I, § 47). For example, he (1937-44/1983) writes: "What, then – does it just twist and turn about within these rules? – It forms ever new rules: is always building new roads for traffic; by extending the network of the old ones" (I, 166); here he is talking about *mathematics*, not about philosophy. More generally, the Wittgensteinian philosopher should not challenge one or another mathematical theorem, but rather help us better to understand its nature and role. He should not form new rules, but rather help us to reach conceptual clarity about the rules such as they are at the present time, in the context of *present* mathematics.

In conclusion, though Putnam's interpretation throws light on Wittgenstein's view of logical necessity, the Putnam-inspired view fails to provide a proper account of Wittgenstein's building of LACs in the philosophy of mathematics.

### 5. *Kantianism and imagination again*

The three interpretations presented in sections II-IV above share the assumption that LACs are radically different from LSCs, and have at least two unwelcome consequences:

- (a) Wittgenstein's building of LACs seems to be in tension with his adoption of a broadly Kantian view of logical necessity.
- (b) The purposes of Wittgenstein's LACs (thus interpreted) seem to disagree with some of Wittgenstein's explicit metaphilosophical claims.

In this section, I will focus on point (a) above, and sketch an alternative view. What is the structure of Wittgenstein's thought experiments? He typically takes into account a familiar or real situation, involving certain familiar and real concepts, languages, and worlds. Then he imagines removing (or changing) one or more features or parameters in that scenario, thus obtaining a different scenario. Nonetheless, the scenario, as modified in imagination, is conceptually and logically *similar* to the initial scenario. Wittgenstein tells us that his "clear and simple language-games [that is, the scenarios modified in imagination] are ... set up as *objects of comparison* which are meant to throw light on the facts of our language by way not only of similarities, but also of dissimilarities" (1953: § 130). In the same vein, he writes: "How do we compare games? By describing them – by describing one as a variation of another – by describing them and *emphasizing* their differences and analogies" (1937-44/1983, II, § 49).

As far as I can tell, Wittgenstein never draws a distinction between LSCs and LACs. In fact, even in the paradigmatic cases of LACs Wittgenstein underlines the *similarities* between bizarre and familiar circumstances. For example, it might be the case that certain people measure with elastic rulers, so that what is called 'measuring' and 'length' is something different from what we call those things. "The use of these words," Wittgenstein tells us, "is different from ours; but it is *akin* to it; and we too use these words in a variety of ways" (1937-44/1983, I, 5).<sup>11</sup> Even the logically strange timber-sellers (who might appear to belong to a "non-economic" form of life) are, in Wittgenstein's view, *somewhat* similar to us. They set the price of an area of wood according to the area it covers, regardless of how much wood the pile contains. Thus, they do not mean the same by "a lot of wood" and "a little wood" as we do. Nonetheless, it is not that they have *no concepts* (of a lot and a little) at all. What they do is not quite paying, calculating, measuring, and perhaps even thinking in our sense, but engaging in a genuine alternative practice that we should call "a quite different system of payment" (1937-44/1983: I, 150; 1939/1976: 202-4; Forster 2004: 112). Perhaps in a similar vein, Wittgenstein points out that "there is something like another arithmetic" (1969: § 375).

<sup>11</sup> Pace Conant, it is a different use, not a mere illusion of use (Forster 2004: 112).



Now, bearing in mind that LACs are as much variations of real or familiar scenarios as LSCs, consider again the idea, outlined in section I above, according to which, in Wittgenstein's LACs, certain imaginary aliens *reject* what we take to be a logically necessary proposition, say  $p$ . This idea should not be interpreted as if such aliens *deny*  $p$ , accept or assent to *not-p*, or even assert *not-p*. Rather, such imaginary people deny  $p^*$ , accept or assert *not-p^\**, where  $p^*$  is *similar* to (but also different from)  $p$ .

On the one hand, it should be emphasized that  $p^*$  should be, at least *prima facie*, very similar (or, at least, similar in many relevant respects) to  $p$ ; otherwise, the imaginary case would be irrelevant for our understanding of the logical (and modal) status of  $p$ . On the other hand, however, there are no grounds for claiming that, strictly speaking,  $p^*$  is necessary or (which is the same, in a broadly Kantian perspective) that *not-p^\** is inconceivable. Therefore, given the premise, according to which Wittgenstein's alleged aliens *deny*  $p^*$ , accept or assent to *not-p^\**, or even assert *not-p^\**, and given the further premise, according to which Wittgenstein accepts the broadly Kantian thesis that  $p$  is logically necessary if we cannot conceive of *not-p*, the controversial conclusion that, according to Wittgenstein, the circumstances described by LACs are ultimately inconceivable and unintelligible, does not follow. Accordingly, the three answers to the question: "For what purpose did Wittgenstein build LACs in his later philosophy?" discussed in this paper (namely, the Conant-inspired answer, the "Cartesian" answer, and the Putnam-inspired answer) are not well justified. In conclusion, under this interpretation, there is no tension at all between Wittgenstein's broadly Kantian view of logical necessity and Wittgenstein's later use of imagination in the philosophy of mathematics. In his LACs Wittgenstein describes not the inconceivable denial of a necessary proposition  $p$  by certain aliens, but rather the (likely) conceivable denial of another proposition  $p^*$ , where  $p^*$  is (relevantly) similar but not identical to  $p$ . In fact, the so-called LACs are not *logically alien* at all. For example, while we are not able to imagine denying  $p$ :  $2+2=4$  (in the ordinary sense of "2", "+" and "4"), we can imagine denying  $p^*$ :  $2+2=4$ , where "+" means not *plus*, but rather *quus* (where 'x quus y' = x plus y if x, y < 4, and = 5 otherwise) (Kripke 1982, though Kripke's own example is slightly different, and is meant to discuss Wittgenstein's most celebrated "alien", namely, the recalcitrant pupil of the *Investigations* §§ 185 ff.);<sup>12</sup> by a similar token, while we are not able to

<sup>12</sup> The reason why this very case is not discussed in this paper is that "Kripkensteinian" problems would have made the issue much more difficult and broad.

imagine denying *p*: *a pile of wood having a base area of 3 square feet and a height of 2 feet contains a larger amount of wood than a pile of wood having a base area of 4 square feet and a height of 1 foot* (where the expression “a larger amount” should be understood in the ordinary sense), we can imagine denying *p\** (which is homo-phonic with *p*), where the expression “a larger amount” means *a larger area*.

## 6. *Metaphilosophy and imagination*

In this section, I shall focus on point (b) of section V above, and show that Wittgenstein’s use of imagination, far from being in tension with his explicit metaphilosophy, somewhat derives from it. Consider, once again, Wittgenstein’s metaphilosophical statements (1953: §§ 126-128): philosophy is merely descriptive; it should simply put everything (that is, our linguistic usage) before us, and neither explain nor deduce anything; the work of the philosopher consists in assembling reminders for a particular purpose; if one tried to advance substantial theses in philosophy, it would never be possible to debate them, because everyone would agree to them. Now, it is natural to ask: what do thought experiments have to do with such a *descriptive* conception of philosophy? *Prima facie*, one might be tempted to answer “Nothing at all: describing linguistic usage is one thing, building imaginary cases is a totally different matter”. On closer inspection, however, it turns out that the use of imagination in philosophy is, in a sense, a natural consequence of a descriptive conception of philosophy. More precisely, thought experiments are the instruments or the techniques that allow the philosopher, as it were, to overcome or avoid the shortcomings of conceiving of philosophy as merely descriptive. My conjecture is that this is how Wittgenstein sees things. He writes: “We can avoid ineptness or emptiness in our assertions only by presenting the model as what it is, as an object of comparison – as, so to speak, a measuring-rod; not as a preconceived idea to which reality *must* correspond. (The dogmatism into which we fall so easily in doing philosophy)” (1953: § 131).

Let me propose an exegesis of this passage. The expression “our assertions” refers to *philosophy* (as Wittgenstein conceives of it). Wittgenstein has a concern: how to avoid ineptness and emptiness in philosophy. Let me begin with ineptness. There are at least two senses in which a philosophical remark can be *inept* (or illegitimate, which is the same) in Wittgenstein’s view. On the one hand, philosophical statements are ille-

gitimate if they are attempts to explain or deduce something, or, more generally, if they are conceived of as substantial theses. Here the source of the illegitimacy is the confusion between philosophy and science (be it a sort of explanatory doctrine such as natural sciences, or a sort of deductive activity such as logic and mathematics). In order to avoid this kind of illegitimacy, a merely descriptive philosophy is more than enough: philosophy as a *mere description* of (the variety of our) linguistic usages, philosophy as linguistic phenomenology, philosophy as a set of insubstantial and/or descriptive “theses” about our language games; philosophy so conceived is perfectly apt (legitimate) in the sense just outlined. Such merely descriptive philosophical statements, however, run the risk of being *empty*: Wittgenstein’s worry is not only about philosophy’s ineptness but also about its emptiness. That is why, he points out, descriptive philosophy requires imagination and thought experiments: the building of imaginary language games is required in order to avoid the risk of emptiness. Philosophy as a mere description of linguistic usage runs the risk of being empty in two senses. First, it does not allow us to discern philosophically interesting features of a puzzle, a situation, or a question from ordinary and non-philosophical features. Secondly, it does not allow us to distinguish between the essential features of a concept and its accidental features (which depend on the idiosyncrasies of ordinary language usages) (Casati 2010: 68). Thus, one of the purposes of the use of imagination in philosophy is to avoid floating off into vacuity. Imagination challenges the flexibility of our concepts, their application in non-ordinary or bizarre circumstances, so that we succeed in seeing what is important or even essential in our concepts. It is no accident that in the very next paragraph of the *Investigations*, § 132, Wittgenstein writes that one of his main purposes is to give “prominence to distinctions which our ordinary forms of language easily make us overlook”.

On the other hand, Wittgenstein warns us not to make a further mistake. When we describe our practices, we have the tendency to consider them as *the* right ones or even as the standard of rightness. This tendency makes our descriptive philosophy *inept* (illegitimate) in a further sense, namely, it makes it dogmatic. In this respect, imagination helps us not to be dogmatic. However, we would run a similar risk of dogmatism even if we use imagination but conceive of it as a mysterious faculty (which nowadays many people would perhaps call “intuition”) that makes it possible to *discover* the given *essence* of reality and/or grammar. That is why we should conceive of our imaginary language games as nothing but *objects of comparison* (1953: § 562): they are just meant to put the flexi-

bility of our concepts to the test, in order to throw philosophical light on our actual language and concepts.

To sum up: description of linguistic usage is an antidote to metaphysics (the illegitimate attempt to provide explanation in philosophy); philosophical imagination is an antidote to merely descriptive philosophy (which runs the risk of being philosophically empty and illegitimately dogmatic); the right interpretation of imaginary cases (as nothing more than objects of comparison) is a further antidote to dogmatism and foundationalism.

For example, it is necessary that  $2 \times 2 = 4$ , but Wittgenstein imagines that certain people believe that twice two is five. For what purposes does he imagine this LAC? First, he attempts to show us that such imaginary people have “a different calculus, or a technique which we should not call ‘calculating’” (1953: II, 227). In other words, he shows us that our concept of calculating is not sufficiently flexible to include the imaginary circumstance: if we stretch that concept too far, we tear it, so that it becomes unrecognizable (= it becomes *another* concept). Secondly, as an anti-foundationalist, he makes us pay attention to the following grammatical fact: though we should not call the imaginary technique “calculating”, we should not consider it as wrong or mistaken either (just as we would not call a coronation “wrong”, even though such a ceremony might look extremely odd to beings very different from ourselves; 1953: II, 227); rather, we should regard it as a different technique.

## 7. Concluding remarks

Wittgenstein accepted a *modified* version of the Kant-Frege-*Tractatus* line of reasoning. On the one hand, he seems to agree that illogical thinking is not thinking at all. For example, he thinks that “logic is *antecedent* to any correspondence between what is said and reality” (1937-44/1983: I, 156). Moreover, he suggests that the laws of logic are the expression of “‘thinking habits’ but also of the habit of *thinking*. That is to say they can be said to shew: how human beings think; and also *what* human beings call ‘thinking’” (I, § 131). In a similar vein, he points out: “The propositions of logic are ‘laws of thought’, ‘because they bring out the essence of human thinking’ – to put it more correctly: because they bring out, or shew, the essence, the technique, of thinking. They shew what thinking is and also shew kinds of thinking” (I, 133). Or: “Logic ... shews us what we understand by ‘proposition’ and by ‘language’” (I, 134). The minimal sense of the above quotations is the Kantian idea, in-

herited by both Frege and the *Tractatus*, that logic is constitutive of thought.

On the other hand, however, Wittgenstein interprets the Kant-Frege-*Tractatus* line of thinking in a novel way. He criticizes Frege, who never provided us with examples and descriptions of logical madness (1937-44/1983: I, §152). He rejects the *Tractatus* view according to which “we could not say what an ‘illogical’ world would look like” (1921-1922/1974: 3.031; 3.032). And he does not take for granted what Kant, Frege and the *Tractatus* took for granted, namely, the idea that the senses of words such as “illogical” and “thinking” are fixed and given once and for all. Not only does he point out that ‘thinking’, ‘understanding,’ and ‘concept’ are vague concepts (1937-44/1983: II, 13; VII, 70), but he also describes certain habits or techniques different from the real ones, in order to check and challenge the flexibility, the resistance, and the boundaries of our concepts of logic, calculus, language, proposition, and thinking. In his view, even though some imaginary cases lack some essential features of what we call ‘thinking’, nonetheless “the line between what we include in ‘thinking’ and what we no longer include in ‘thinking’ is no more a hard and fast one than the line between what is still and what is no longer called ‘regularity’” (I, 116).

As I have tried to show in this paper, several of Wittgenstein’s philosophical thought experiments might be read as attempts to throw light on the elastic boundary lines between what we include under a certain concept (say, thought) and what we no longer include under it: perhaps one of Wittgenstein’s most famous imaginary cases, namely, the builders’ language game described in the *Investigations* § 2, too, could be regarded, to a large extent, as a way of establishing *how much logic* – in one or another sense of “logic” – we need to speak properly of “speaking and understanding a language” and “thinking”.

Generally speaking, such thought experiments bring us to one or the other side of such boundary lines, or, if you like, they keep on drawing those lines again and again, so that we learn something new about such lines, such concepts, and ourselves as concept-users.<sup>13</sup>

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## References

- Alanen, Lilli, 1991, "Descartes, Conceivability, and Logical Modality", in Tamar Horowitz (ed.), *Thought Experiments in Science and Philosophy*, Rowman and Littlefield, Lanham.
- Aquinas, Saint Thomas, 1265-1274/1947, *Summa theologiae*, engl. transl. by Fathers of the English Dominican Province, Benziger Bros., New York.
- Broyles, James E., 1974, "An Observation on Wittgenstein's Use of Fantasy", *Metaphilosophy*, 5: 291-297.
- Canfield, John, 1975, "Anthropological Science Fiction and Logical Necessity", *Canadian Journal of Philosophy*, 4: 467-479.
- Casati, Roberto, 2010, *Prima lezione di filosofia*, Laterza, Roma-Bari.
- Coliva, Annalisa, 2010, "Was Wittgenstein an Epistemic Relativist?", *Philosophical Investigations*, 33: 1-23. <http://onlinelibrary.wiley.com/doi/10.1111/j.1467-9205.2009.01394.x/pdf>.
- Conant, James, 1991, "The search for logically alien thought: Descartes, Kant, Frege, and the *Tractatus*", *Philosophical Topics*, 20: 115-180.
- Descartes, René, 1648/1984-91, *The Philosophical Writings of Descartes*, 3 vols., engl. transl. by J. Cottingham, R. Stoothoff, D. Murdoch, and A. Kenny, Cambridge University Press, Cambridge.
- Einstein, Albert, 1917/1961, *Relativity: The Special and General Theory: A Popular Exposition*, Grown Publishers, New York.
- Forster, Michael N., 2004, *Wittgenstein on the Arbitrariness of Grammar*, Princeton University Press, Princeton.
- Frege, Gottlob, 1884/1984, *The Foundations of Arithmetic*, engl. transl. by J.L. Austin, Blackwell and Mott., New York.
- Frege, Gottlob, 1893-1903/1982, *The Basic Laws of Arithmetic: Exposition of the System*, engl. trans. by Montgomery Furth, University of California Press, Berkeley.
- Geach, Peter, 1977, *Providence and Evil*, Cambridge University Press, Cambridge.
- Gendler, Tamar and Hawthorne, John (eds.), 2002, *Conceivability and Possibility*, Oxford University Press, Oxford.
- Glock, Hans-Johann, 2002, "'Clarity' is not Enough", in R. Haller and K. Puhl (eds.), *Wittgenstein and the Future of Philosophy: Proceedings of the 24th International Wittgenstein Symposium*, Hölder-Pichler-Tempsky, Vienna: 81-98.
- Glock, Hans-Johann, 1997, "Kant and Wittgenstein: Philosophy, Necessity and Representation", *International Journal of Philosophical Studies* 5 (2): 285-305.

- Horsten, Leon, "Philosophy of Mathematics", *The Stanford Encyclopedia of Philosophy*, 2012, E.N. Zalta (ed.). <http://plato.stanford.edu/archives/sum2012/entries/philosophy-mathematics/>
- Janik, Allan, 2011, "Review of Klagge, James Carl, *Wittgenstein in Exile*", in Habsburg, H-Net Reviews. <http://www.h-net.org/reviews/showrev.php?id=32492>
- Kant, Immanuel, 1781-87/1999, *Critique of Pure Reason*, Engl. Transl. by Paul Guyer and Allen Wood, Cambridge University Press, Cambridge.
- Kant, Immanuel, 1800/1992, "The Jasche Logic", in J.M. Young (ed.), *Lectures on Logic*, Cambridge University Press, Cambridge.
- Kripke, Saul, 1982, *Wittgenstein on Rules and Private Language*, Blackwell, Oxford.
- Lear, Jonathan. 1984, "The Disappearing 'We'." *Proceedings of the Aristotelian Society*, Supplementary Volume 58: 219-258.
- MacFarlane, John, 2002, "Frege, Kant, and the Logic in Logicism", in *The Philosophical Review*, 111: 25-65. [http://secure.pdcnet.org/phr/content/phr\\_2002\\_0111\\_0001\\_0025\\_0066](http://secure.pdcnet.org/phr/content/phr_2002_0111_0001_0025_0066)
- Marconi, Diego, 2010, "Wittgenstein and Necessary Facts", in Pasquale Frascolla, Diego Marconi and Alberto Voltolini (eds.), *Wittgenstein: Mind, Meaning and Metaphilosophy*, Palgrave Macmillan, London: 139-65.
- Peach, Andrew, 2004, "The Origins of Wittgenstein's Imaginary Scenarios: Something Old, Something New", in *Philosophical Investigations*, 27: 299-327. <http://onlinelibrary.wiley.com/doi/10.1111/j.1467-9205.2004.00229.x/abstract>
- Penco, Carlo, 2010, "The Influence of Einstein on Wittgenstein's Philosophy", in *Philosophical Investigations*, 33: 360-379. <http://onlinelibrary.wiley.com/doi/10.1111/j.1467-9205.2010.01415.x/abstract>
- Putnam, Hilary, 1992, "Reply to James Conant", in *Philosophical Topics*, 20: 374-377.
- Putnam, Hilary, 1994, "Rethinking Mathematical Necessity", in Putnam, Hilary, *Words and Life*, ed. James Conant, Harvard University Press, Cambridge, Mass.: 245-263.
- Reck, Erich, 2005, "Frege on Numbers: Beyond the Platonist Picture", in *Harvard Review of Philosophy*, 13: 25-40.
- Stroud, Barry, 1965, "Wittgenstein and Logical Necessity", in *Philosophical Review*, 74: 504:18.
- Williams, Bernard 1974. "Wittgenstein and Idealism". In *Understanding Wittgenstein*, ed. Vesey G.N.A., London: McMillan.
- Williams, Meredith, 2010, "Method and Metaphilosophy in the Philosophical Investigations", in Pasquale Frascolla, Diego Marconi and Alberto Voltolini (eds.), *Wittgenstein: Mind, Meaning and Metaphilosophy*, Palgrave Macmillan, London: 229-251.

- Wittgenstein, Ludwig, 1921-1922/1974, *Tractatus Logico-Philosophicus*, D. Pears and B. McGuinness (eds.), Repr. Routledge, London and New York.
- Wittgenstein, Ludwig, 1929-1930/1975, *Philosophical Remarks*, ed. R. Rhees, Blackwell, Oxford.
- Wittgenstein, Ludwig, 1929-1932/1967, *Wittgenstein und der Wiener Kreis*, notes from Friedrich Waismann, ed. B. McGuinness, Blackwell, Oxford.
- Wittgenstein, Ludwig, 1933-1935/1958, *The Blue and Brown Books*, Blackwell, Oxford.
- Wittgenstein, Ludwig, 1937-1944/1983, *Remarks on the Foundations of Mathematics*, ed. G.H. von Wright, R. Rhees, E. Anscombe. Repr. The MIT Press, Cambridge MA.
- Wittgenstein, Ludwig, 1939/1976, *Lectures on the Foundations of Mathematics*, ed. C. Diamond, Cornell UP, Ithaca.
- Wittgenstein, Ludwig, 1953/1958, *Philosophical Investigations*, ed. E. Anscombe, R. Rhees. Repr. Blackwell, Oxford.
- Wittgenstein, Ludwig, 1967, *Zettel*, ed. E. Anscombe, G.H. von Wright, Blackwell: Oxford.
- Wittgenstein, Ludwig, 1969, *On Certainty*, Ed. E. Anscombe and G.H. von Wright, Blackwell, Oxford.
- Wittgenstein, Ludwig, 2000, *Nachlass*, Bergen Electronic Edition, Oxford University Press, Oxford.
- Wittgenstein, Ludwig, 2005, *The Big Typescript*, TS 213. Ed. and Engl. trans. C.G. Luckhardt and M.A.E. Aue. Blackwell, Oxford.