A language for the human body: a tentative proposal

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Abstract: To rigorously describe the structure of the human body a rich formal language is needed: this language must be able to describe all the parts of the body and the spatial regions those parts occupy; moreover, it must be able to describe the relations that occur between those parts and those spatial regions; finally, it must be able to distinguish between essential and contingent features of the body and it must do so based on the context that is relevant for the descriptions given. Our aim in this paper is to provide a formal language that can express all those kinds of information. The language we present is inspired by Vakarelov (2008) and is a modally augmented version of the discrete mereotopology due to Galton (2014) with an added relation for location (this latter addition is inspired by Donnelly (2004)): we will call this language *modal discrete mereotopology with location*. In the paper, we also suggest a neighbourhood semantics for our language: this will make the language context-sensitive, making it fit for different computer graphics applications.

Keywords: Human body; modal mereotopology; discrete mereotopology; location.

1. *Introduction*

The problem we try to solve in this paper is that of providing a formal language rich enough to rigorously describe the structure of the human body. The idea that it is possible to rigorously capture and describe features of the human body is an ancient one: many studies on the proportions between its parts can be found in writings concerning ancient Greek statues, e.g. Polyklòeitos' *Kanon*. We can also find many developments of the idea through the centuries, e.g. Luca Pacioli's *De Divina Proportione* or Leonardo Da Vinci's mathematical studies of the human body and its parts; one remarkable example of the advances of such studies is Vitruvius' *De Architectura*:

For the human body is so designed by nature that the face, from the chin to the top of the forehead and the lowest roots of the hair, is a tenth part of the whole height; the open hand from the wrist to the tip of the middle finger is just the same; the head from the chin to the crown is an eighth, and with the neck and shoulder from the top of the breast to the lowest roots of the hair is a sixth; from the middle of the breast to the sum-

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mit of the crown is a fourth. If we take the height of the face itself, the distance from the bottom of the chin to the under side of the nostrils is one third of it; the nose from the under side of the nostrils to a line between the eyebrows is the same; from there to the lowest roots of the hair is also a third, comprising the forehead. The length of the foot is one sixth of the height of the body; of the forearm, one fourth; and the breadth of the breast is also one fourth. The other members, too, have their own symmetrical proportions, and it was by employing them that the famous painters and sculptors of antiquity attained to great and endless renown. (Vitruvius 1914: Book 3; Chapter 1; Section 2).

In contemporary times, with the development and the diffusion of digital technologies, more and more researches have been made on the topic of mathematical models for the structure of the human body, generating a multitude of formal languages trying to achieve a solution to the problem of rigorously describing the human body.[1] Those languages are used especially in the field of computer graphics and our aim in this paper is to provide a language for such applications. Given our aim, we must meet four desiderata: our formal language should be: 1) able to give both a micro- and a macro-description of the human body; 2) fine grained enough to describe all the parts of the body and their relationships; 3) able to capture essential features of the structure of the body; 4) able to distinguish different contexts of description.

Specifically, the language we present is inspired by Vakarelov (2008) and is a modally augmented version of the discrete mereotopology due to Galton (2014) with an added relation for location (this latter addition is inspired by Donnelly (2004)): we will call this language *modal discrete mereotopology with location* (MDML). The basic idea is that MDML has the syntactic strength to wholly describe the structure of the human body. The two major advantages of our approach are related to the choice of using discrete mereotopology as the starting point and the use of neighbourhood semantics to interpret the modal operators (Scott 1970; Montague 1970; Chellas 1980):[2] those choices allow MDML both to be practically useful from a computer science modelling point-of-view and to be context sensitive when describing the structure of the human body. The paper will proceed as follows: in section two, we introduce the basic language with which we will work along the paper; in section three, we describe the two main relations (part and connection) that characterize our language; in section four, we describe the location function and explain what benefit we get from admitting this special function in our language; in section five, we augment the previously presented language with modal operators and we give for them a neighbourhood semantics that serves as

[1] See for example Zinkovsky *et al.* (1996); Ottesen *et al.* (2004) and Bastioni & Graziani (2011).

[2] For further indications on neighbourhood semantics see Pacuit (2014) and Zhuang-hu (ts).

our interpretation of the operators; in section six, we show how our language meets our desiderata.

Before we proceed, we need to make a few methodological remarks to clarify our approach to the problem we posed. The first methodological remark is that we often refer to the literature of qualitative spatial reasoning; this is because we believe this is the field that can benefit the most from our insights. The second remark is that this paper is the production of a work-in-progress project, therefore some key details (such as the metatheorems for the theory) are missing. The third and final remark is that we will not provide any algorithm or complete example of the application of our approach: this choice stems from the fact that our work is still in a development phase and we rather focus on the core ideas that influenced our approach.

We can now proceed to the next section, where we present the language that builds up the core of our approach.

2. *The syntax of MDML*

The syntax of our language is basically that of first-order modal logic with identity, where the specific features of MDML are the presence of:

i. Two distinct countable sets of constant symbols $\boldsymbol{t}_1,\ldots, \boldsymbol{t}_n$ for objects and $t_1,\ldots, t_n$ for spatial regions;[3]
ii. One countable set of variables x^{obj}, y^{obj}, z^{obj},… with eventual subscripts, ranging over *Obj*.
iii. One countable set of variables x^U, y^U, z^U,… with eventual subscripts, ranging over $\wp U \backslash \varnothing$;
iv. Two special binary relation symbols *P* and *C* for *part* and *connection*;
v. One special function symbol *r* for *location*;

Terms, formulas and sentences are recursively defined in the standard way. The only special feature we require from our grammar is that if there are two distinct variables saturating a predicate, then those two variables must range

[3] We will indicate the domain of objects with *Obj* and the domain of spatial regions with $\wp U$ (powerset of *U*), where *U* is a set containing cells, i.e. basic atomic components of spatial regions. We take *Obj* to consist of both material (e.g. noses) and immaterial (e.g. holes) objects. We only exclude spatial regions from the domain *Obj*. We are conscious that spatial regions might indeed be seen as objects in them-selves, but we assume, for the purpose of the paper, that they are not objects. Finally, we will exclude the empty set from the powerset of *U* and every occurrence of the powerset symbol in our paper will stand for the powerset minus the empty set.

over the same domain, i.e. either *Obj* or $\wp U$.[4] This amounts to say that we do not allow inter-domain saturation of a predicate. The same applies to saturation through constants and general terms.

In the rest of the paper, we are going to employ variable symbols (x, y, z, …) without superscripts; those variables should be thought of as standing for either the object variables or the spatial region variables, we will specify the variable's superscript where the distinction is significant.

In the next section, we will give some insights on the special relations P and C.

3. *The language of discrete mereotopology*

To define the meaning of our two predicates P and C we will make use of a version of discrete mereotopology inspired by Galton (2014). To discuss the properties of the predicates we will follow an axiomatic approach; this helps us in staying closer to the literature about mereotopology and makes the discussion clearer. The section proceeds in three steps: in step one, we give the formal structure of discrete mereotopology; in step two, we give an interpretation of the predicates that is inspired by Galton (2014); in step three, we give an example that shows how the fragment of our language where only the two predicates P and C are used can capture some information about the structure of the human body.

3.1. *The formal structure*

Mereotopology is the theory of the relations of part and connection. Simplifying, mereotopology is a merging theory, where the two simpler theories that are merged together are mereology (the theory of parts and wholes) and topology (the theory concerned with the study of some properties of space). Specifically, the version of mereotopology we discuss in this paper is inspired by Antony Galton's discrete mereotopology (Galton 2014), whose starting point is *Extensional Mereology* (EG; Varzi 2016), to which axioms for atomicity and a basic notion of connection (interpreted on an *adjacency space*) are added. The resulting theory – a special case of the *Region Connection Calculus* – is a powerful language that has already many of the desirable features of an ideal formal language that can describe the human body. In this subsection,

[4] It should be pointed out that this choice is a technical one that help us in dealing with some theoretical problems that arise when some of the axioms we will introduce range over distinct domains. Moreover, we are aware that other type of solutions to those problems might be provided. We choose to follow this path because we believe this is the simplest and most understandable between all the available ones.

we introduce the formal structure of the mereotopological theory we are going to work with.

The mereotopological theory we are working with assumes two primitive relations P (informally interpreted as *part*) and C (informally interpreted as *connection*). P is a partial order, i.e. a reflexive, transitive and antisymmetric relation:[5]

(Ax. 1) $P(x,x)$
(Ax. 2) $P(x,y) \land P(y,z) \rightarrow P(x,z)$
(Ax. 3) $P(x,y) \land P(y,x) \rightarrow x=y$[6]

(Ax. 1) says that every object is part of itself; (Ax. 2) says that if a first object is part of a second object, and this second object is, in turn, part of a third one, then the first one is part of the third one; (Ax. 3) says that if two objects are respectively one part of the other, then they end up being one and the same object. From this basic notion of part, we can then introduce, through definition, different new relations. We will be interested in the relations of: (D. 1) *proper part*; (D. 2) *overlap*; (D. 3) *underlap* and (D. 4) *being disjoint*.

(D. 1) $PP(x,y)=_{def} P(x,y) \land \neg P(y,x)$
(D. 2) $O(x,y)=_{def} \exists z(P(z, x) \land P(z,y))$
(D. 3) $U(x,y)=_{def} \exists z(P(x, z) \land P(y,z))$
(D. 4) $DR(x,y)=_{def} \neg O(x,y)$

Before moving on to the notion of connection, some further reflection is needed on (Ax. 1) – (Ax. 3) and how those axioms capture our intuitive idea of the notion of part. (Ax. 1) – (Ax. 3) are not sufficient to univocally identify and completely define the notion of part. To make an example, the relation *less-than-or-equal-to* defined over natural numbers is also a partial order; however, *less-than-or-equal-to* is not the same relation as that of part. Therefore, to univocally identify the notion, we need to add more to the three axioms stated above. One possibility is to add what in the literature is called *strong supplementation*. Strong supplementation is a member of the family of decomposition principles. Those principles govern the relations that hold between the whole on the one side and its parts on the other. Strong supplementation is commonly used to capture the intuition that no whole can have a single proper part (what

[5] (Ax. 1) – (Ax. 3) are the core axioms of every mereological theory and the theory which comprises them all is called *Ground Mereology*.

[6] We assume that every formula given is closed under universal quantification for each of the variables that appear in the formulas.

strong supplementation says is that any object which is not part of a second object, must, at least, have a part that doesn't overlap with that second object).[7]

(Ax. 4) $\neg P(y,x) \rightarrow \exists z(P(z,y) \land \neg O(z,x))$

To see how (Ax. 4) helps in identifying better the relation of part, let's go back to our comparison between the relation *less-than-or-equal-to* (≤) and that of *part*; as we have seen above, ≤ is a partial order just like the relation of part as defined by (Ax. 1) – (Ax. 3). This means that a theory that considers only (Ax. 1) – (Ax. 3) can't distinguish between the two notions. However, if we add (Ax. 4) to the theory, we avoid such problem. If fact, (Ax. 4) does not hold for ≤, but it does for the relation of part.

The mereological theory characterized by (Ax. 1) – (Ax. 4) is what Varzi (2016) dubbed *Extensional Mereology* (EG). The label extensional comes from the fact that in this theory you can prove that having the same parts is a necessary and sufficient condition for being the same object. EG is Galton's (2014) starting point. However, we will add a further axiom taken from the class of composition principles (i.e. principles that govern the relation between parts, on the one side, and the whole, on the other). We do so because a formal language that is employed to describe the human body must be able to talk about how different parts can form different, composite objects. The new axiom, i.e. *unrestricted composition*, is based on the notion of *mereological fusion*.

(D. 5) $F(z,\varphi(w)) =_{\mathrm{def}} \forall v(O(v,z) \leftrightarrow \exists w(\varphi(w) \land O(w,v)))$

Where z is the mereological fusion of the w with property φ. Remember that all the variables that appear in the above definition must have the same range, i.e. either *Obj* or $\wp U$.[8] This requirement, i.e. that the variables z, w and v all range over the same domain, guarantees that the fusion of objects is itself an object and the fusion of spatial regions is itself a spatial region.

From this principle, follows unrestricted composition:

(Ax. 5) $\exists w(\varphi(w)) \rightarrow \exists z(F(z,\varphi(w)))$

The theory we obtain with (Ax. 1) – (Ax. 5) is called *General Extensional Mereology* (GEM).[9]

[7] This intuition can be captured in different ways, each of which has different strength (i.e. admit of different classes of models). Again, our choice of using strong supplementation is driven by our desire to remain close to Galton's theory.

[8] Note that this means we do not allow from composite entities made both of objects and spatial regions.

[9] See Varzi (2016) for further insights on GEM.

We can now define C. C is a reflexive and symmetric relation:

(Ax. 6) $C(x^U, x^U)$
(Ax. 7) $C(x^U, y^U) \rightarrow C(y^U, x^U)$

Moreover, the minimal relation that holds between P and C is the following:

(Ax. 8) $P(x, y) \rightarrow \forall z^U (C(z^U, x^U) \rightarrow C(z^U, y^U))$[10]

C should be informally interpreted as the relation of *connection*.[11]

With the addition of C, we can now introduce, as we did previously, different new relations. We will introduce the relations of: (D. 6) *disconnection*; (D. 7) *external connection*; (D. 8) *tangential part* and (D. 9) *non-tangential part*:

(D. 6) $DC(x^U, y^U) =_{def} \neg C(x^U, y^U)$
(D. 7) $EC(x^U, y^U) =_{def} C(x^U, y^U) \wedge \neg O(x^U, y^U)$
(D. 8) $TP(x^U, y^U) =_{def} P(x^U, y^U) \wedge \exists z^U (C(x^U, z^U) \wedge \neg O(z^U, y^U))$
(D. 9) $NTP(x^U, y^U) =_{def} P(x^U, y^U) \wedge \forall z^U (C(x^U, z^U) \rightarrow O(z^U, y^U))$[12]

(Ax. 1) – (Ax. 8) (except (Ax. 5) which is not present in standard mereotopologies) form the base of every mereotopological theory.[13] To this base, we will add a further axiom, which will impose on our theory that the domain on which the terms vary is discrete. This will turn out to be fundamental for the employability of our language for computer graphics purposes. To introduce such axiom, we define a new monadic predicate: (D. 10) being an atom.

(D. 10) $Atom(x) =_{def} \neg \exists y (PP(y,x))$

(Ax. 9) states that entities called atomic parts exist and that they form the base that build up objects.

(Ax. 9) $\exists y (Atom(y) \wedge P(y,x))$

[10] For clarity and completeness, we anticipate here something that we will introduce in the paper later, i.e. the location function: the two variables x^U and y^U stand specifically for the two spatial regions associated with the variables x and y. In order to get this association, a location function must be applied to the variables, i.e. $x^U = r(x)$ and $y^U = r(y)$

[11] Even though it is in principle possible to make an analysis of the notion of connection similar to the one we made previously of the notion of part, for the purpose of this paper, no further requirements will be necessary to capture the notion of connection. We will therefore limit ourselves to (Ax. 6) – (Ax. 8).

[12] Given our definitions above, it follows that TP and NTP are mutually exclusive.

[13] We shall repeat that the idea expressed by (Ax. 4) can be expressed in different ways, with different strengths.

(Ax. 1) – (Ax. 9) are the axioms of our discrete mereotopological theory and they are all we need to proceed with our discussion.[14]

3.2. *The interpretation*

In subsection 3.1 we introduced all the axioms that form the mereotopological theory we will use in successive sections. In what follows, we provide an interpretation of the predicates we introduced in the previous subsection. This interpretation is owed to Galton (2014).

Galton introduces, by definition, a general class of entities, i.e. *adjacency spaces*.

(D. 11) An *adjacency space* is a non-empty set U of entities called *cells* together with a reflexive, symmetric relation $\sim \subseteq U \times U$, called *adjacency*.[15]

Cells are to be considered as spatial entities that are elements of our domain U. We will refer to cell with the symbols $c_1, \dots, c_n$. Additionally, over an adjacency space, it is possible to define other entities, called *regions*, which represent aggregates of cells, i.e. subsets of U. Keep in mind that it is not required that a region is made up of two or more cells, leaving open the possibility of having one-cell regions. However, it remains important to have a distinction between cells and regions. This means that a cell and the region made up of only that cell are two *conceptually* distinct entities.[16]

Given the theory defined in subsection 3.1 and an adjacency space, an interpretation I (given in set-theoretical terms) of the theory of discrete mereotopology over an adjacency space (U) is specified as follows:

- Each individual term t of the theory denotes a non-empty subset of the adjacency space, i.e. $t^{\mathrm{I}} \subseteq \boldsymbol{U}$.[17]
- A relation $P(t_1, t_2)$ is interpreted to mean that $t_1^{\mathrm{I}} \subseteq t_2^{\mathrm{I}}$.
- A relation $C(t_1, t_2)$ is interpreted to mean that there are cells $c_1 \in t_1^{\mathrm{I}}$ and $c_2 \in t_2^{\mathrm{I}}$ such that $c_1 \sim c_2$.

[14] As shown in Varzi (2014), (Ax. 4) and (Ax. 9) can be merged into a single axiom. We prefer to keep them distinct, to simplify the discussion that will come later in this paper.

[15] Galton 2014: 298, emphasis in original. The reader might well suspect, reading this definition, that the interpretation of our theory will be given in set-theoretical terms

[16] Examples of similar practices can be found in set theory, where an element and the singleton set which contains only that element are two conceptually distinct entities.

[17] It must be pointed out that terms refer to regions and not to cells. Cells are introduced only when we define adjacency spaces and aid us in dealing with the interpretation of the connection relation. Moreover, as we said, they are what compose regions. Informally, cells are basic primitive entities, among which the only relation that can exist is that of adjacency. However, since mereotopology is interested only in the relations of part and of connection, this theory doesn't talk at all about cells, but only about regions.

Two regions are connected if, and only if, the adjacency relation holds for two cells, one taken from one region and the other taken from the other region.[18] All the formulas of the theory presented in subsection 3.1 are to be interpreted according to this interpretation.

We should take care in pointing out that, up until now, we did not specify how the adjacency relation is defined. This is because the general version of the adjacency relation is underdefined, i.e. to obtain a clear definition we must specify exactly what *U* is and what informal idea of adjacency we have in mind. In this paper, we take *U* to be the set $\mathbb{Z}^2$, which is the algebraic domain of two-dimensional discrete cells; moreover, our cells will have a square shape and we intend two cells to be adjacent if, and only if, they share an edge (we do not count two cells sharing *only* a vertex as adjacent cells). It is now possible to give a definition for a particular $\sim_4$, which is the adjacency relation for squares on $\mathbb{Z}^2$:[19]

$$(\boldsymbol{x}, \boldsymbol{y}) \sim_4 (\boldsymbol{x}', \boldsymbol{y}') \text{ iff } |\boldsymbol{x}-\boldsymbol{x}'| + |\boldsymbol{y}-\boldsymbol{y}'| \leq 1$$

It is obviously possible to define similar adjacency relations for different domains, e.g. $\mathbb{Z}^3$ or $\mathbb{R}^3$ and different informal adjacency relations, e.g. $\sim_8$ for the adjacency relation holding between squares that share either a side or a vertex. Even though it is easier to define the adjacency relation for shapes like squares or hexagons (which lack a preferential orientation), it is also possible to define it for shapes like triangles. In this case, disjunctive definitions are needed, where the two disjoints capture the two possible orientations of the triangles (pointing upwards or pointing downwards). For example, $\sim_3$, which captures the informal adjacency relation between triangles that share an edge (but not a vertex), can be defined as follows: an up-pointing triangle $(\boldsymbol{x}, \boldsymbol{y})$ is three-adjacent to $(\boldsymbol{x}, \boldsymbol{y}-1)$, $(\boldsymbol{x}, \boldsymbol{y}+1)$ and $(\boldsymbol{x}-1, \boldsymbol{y}+1)$, while a down-pointing triangle $(\boldsymbol{x}, \boldsymbol{y})$ is three adjacent to $(\boldsymbol{x}, \boldsymbol{y}-1)$, $(\boldsymbol{x}, \boldsymbol{y}+1)$ and $(\boldsymbol{x}+1, \boldsymbol{y}-1)$.[20]

As we will show in detail in the next section, it is common, in computer graphics, to have cells with a defined shape (i.e. square shape) and this is the reason we gave the definition of adjacency relation above.

[18] Note that the adjacency relation holds between cells, while the connection relation holds between regions.

[19] We employ boldface variables to refer to numerical coordinates on $\mathbb{Z}^2$

[20] We are indebted to Galton (personal communication) for this definition of the three-adjacency relation for triangles.

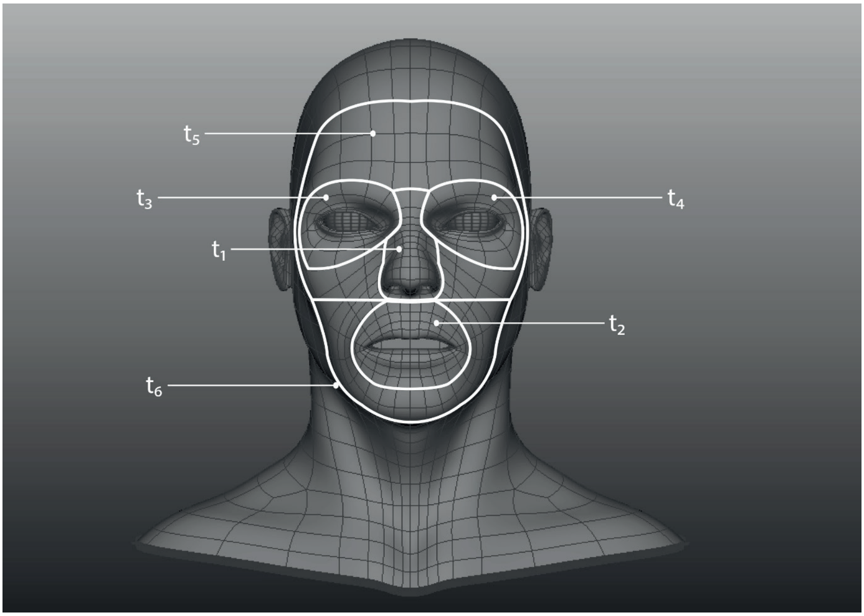

Fig. 1. A face with six spatial regions indicated.

3.3. *Scenario*

We now provide a scenario for the application of the fragment of MDML – where only *P* and *C* are present – to a human face in a possible computer graphic software application. This should help the reader understand the benefits of having those two predicates as special relations in our language.

Suppose we have a face (fig. 1) on which we identified six spatial regions: t_1: the nose; t_2: the lips; t_3: the right eye; t_4: the left eye; t_5: the upper part of the head and t_6: the whole face. Those six spatial regions are formed by squared cells that are members of *U*. We can then use the fragment of MDML we introduced in the previous subsections to describe some properties of the structure of the face. For instance, we can say that:

$C(t_1, t_2) \wedge C(t_1, t_3) \wedge C(t_1, t_4)$[21]

But

$DC(t_2, t_3) \wedge DC(t_2, t_4) \wedge DC\ (t_3, t_4)$

[21] We use here the connection predicate to keep the example simple. Obviously, the modeler can make use of stronger relations, such as that of tangential part or external connection.

The graphic designer will also be able to indicate parthood relations among regions:

$PP(t_1, t_5) \wedge PP(t_3, t_5) \wedge PP(t_4, t_5) \wedge PP(t_5, t_6)$

But

$DR(t_2, t_5)$

Once some relations are fixed by the modeller, other relations can be derived automatically following the axioms and using some simple rules such as *Modus Ponens*. An example is:

$PP(t_1, t_6)$

which can be derived from the two relations: $PP(t_1, t_5)$ and $PP(t_5, t_6)$.

We now move to the next section, where we discuss the meaning and behaviour of the function r of MDML and show how this function can increase the quality of a description of the structure of the human body.

4. *The location function*

We will now discuss the meaning of the function r. Informally the function r is a location function that associates a unique region of space with every object of our domain. The way we interpret the function and the properties we associate with it are due to Donnelly (2004). As for section 3, we will continue to present the properties of r in an axiomatic form. The section proceeds in two steps: in step one, we add the axioms governing r to the fragment of MDML we already presented in section 3 and we extend our interpretation based on adjacency spaces so that r receives a proper interpretation; in step two, we show how this new function increases the amount of information we can capture expanding the scenario we made at the end of section 3.

4.1. *Adding location to the picture*

It is important to note that the theory presented in section 3, coupled with its interpretation, manages only to talk about regions' spatial properties and not about objects' spatial properties, an example of the latter being that of spatial coincidence between two objects which do not overlap.[22] To obtain

[22] Imagine, for example, a child that puts one of his fingers in his nose. In that case, we would have two objects (the finger and the nose) that share a spatial region, even though the two are quite distinct objects.

such a fine-grained description, we introduce a primitive function symbol *r*, which takes as arguments objects from the domain $\wp Obj$ and returns as values the objects' unique spatial regions from $\wp U$. We use the power set of *Obj* instead of just *Obj* to allow uniformity in the interpretation of the theory we present. This amount to the philosophical thesis that sees composite objects as sets containing their parts. This will allow us to talk about objects that share a common location without overlapping. The axioms describing the behaviour of the function *r* are the following:

(Ax. 10) $\forall x \exists y^{U}(y^{U}= r(x))$
(Ax. 11) $P(x^{U}, y^{U}) \rightarrow P(r(x), r(y))$[23]

Informally (Ax. 10) states that every entity has an associated region of space[24] and (Ax. 11)[25] states that if an object is part of another object, then the first object's associated spatial region is a part of the second object's associated spatial region.[26] The introduction of the location function allows to define further interesting relations: (D. 12) partial coincidence and (D.13) located-in.

(D. 12) $PC(x, y)=_{def} O(r(x), r(y))$
(D. 13) $LI(x, y)=_{def} P(r(x), r(y))$

This latter addition is very useful when it comes to the representation of holes or of objects lying in holes.[27] Indeed, the main reason that guided our intention to include location in our language was that of keeping track of holes in the body.

It is important to note that, to achieve our goal of having a language expressive enough to distinguish between objects and the spatial region they occupy, the location function *r* alone isn't enough. Much of this expressive power comes from the fact that we included two distinct domains *Obj* and

[23] Another possibility for adding location is that followed by Donnelly (2003). Donnelly suggests to modify the semantical level (i.e. the model of interpretation) to allow layers. In this case objects might occupy different layers of which one has the special status of region layer. At that point, the function *r* just assigns to each member of the domain of objects its representative in the region layer.

[24] If $x \in \wp U$, i.e. the entity to which the location function is applied is a spatial region, then $r(x^{U})=x^{U}$, i.e. the spatial region associated with x^{U} is x^{U} itself.

[25] If the variables that fall under the scope of the location function range over the domain $\wp U$, then (Ax. 11) becomes the trivial identity axiom of propositional logic, i.e. $P(x^{U}, y^{U}) \rightarrow P(x^{U}, y^{U})$.

[26] It is important to remark that the spatial regions that *r* associates with objects are discrete spatial regions made up of cells. Objects can therefore be seen as occupants of those spatial regions that are defined in terms of the adjacency spaces. For a thorough investigation of this idea see Donnelly 2003.

[27] This is what Donnelly (2004) tries to achieve.

$\wp U$ in our language. What the location function does is put those two domains in relation, i.e. r is a function from the power set of *Obj* to the power set of *U* (formally: $r : \wp Obj \mapsto \wp U$). We use the power set of *U* as the range of the function because we assume that objects always occupy a region of space and never a single cell of *U*.[28]

Note also that for the two primitive relations P and C we talked about in section 3, we allow only P to be applied to both $\wp Obj$ and $\wp U$, while C will always be properly applied to regions of space (i.e. elements of $\wp U$); we will informally talk about *C* holding between elements of $\wp Obj$, but we will mean that *C* holds between the regions of space that those elements are associated with according to *r*. Moreover, remember that the terms that saturate a predicate must always be taken from the same domain, e.g. if we have $P(x,y)$, both *x* and *y* must range over the same domain (either *Obj* or $\wp U$).

Having the extra domain *Obj* in the language, requires an extension of the interpretation we gave in subsection 3.2 that can deal with this new domain.

The interpretation *I* is extended for the two relations over the domain *Obj* as follows:

- Each individual term ℓ of the theory denotes a non-empty subset of the set of objects, i.e. $\ell^I \subseteq Obj$.
- A relation $P(\ell_1, \ell_2)$ is interpreted to mean that $\ell_1^I \subseteq \ell_2^I$.
- A relation $C(\ell_1, \ell_2)$ is interpreted to mean that $C(t_1, t_2)$ holds between $t_1=r(\ell_1)$ and $t_2=r(\ell_2)$.

All the details concerning this extension are equivalent to the ones concerning the interpretation given in 3.2, therefore we will avoid repeating them.

We now show how this strengthened language can help improving the quality of the description of the human body, making use of our previous example of the face.

4.2. *Scenario extended*

Suppose we start with the face of (Fig. 1) and we identify three more peculiar objects on it (obtaining Fig. 2): ℓ_7: the mouth's hole; ℓ_8: the right naris and ℓ_9: he left naris. Those three objects are members of $\wp Obj$. Therefore, we can apply the *r* function to those objects and obtain their unique spatial region:

$$(r(\ell_7)=t_7) \wedge (r(\ell_8)=t_8) \wedge (r(\ell_9)=t_9)$$

[28] In fact, if it happens that an object occupies only a single cell of *U*, we solve this by saying that the object occupies the region made of only that cell.

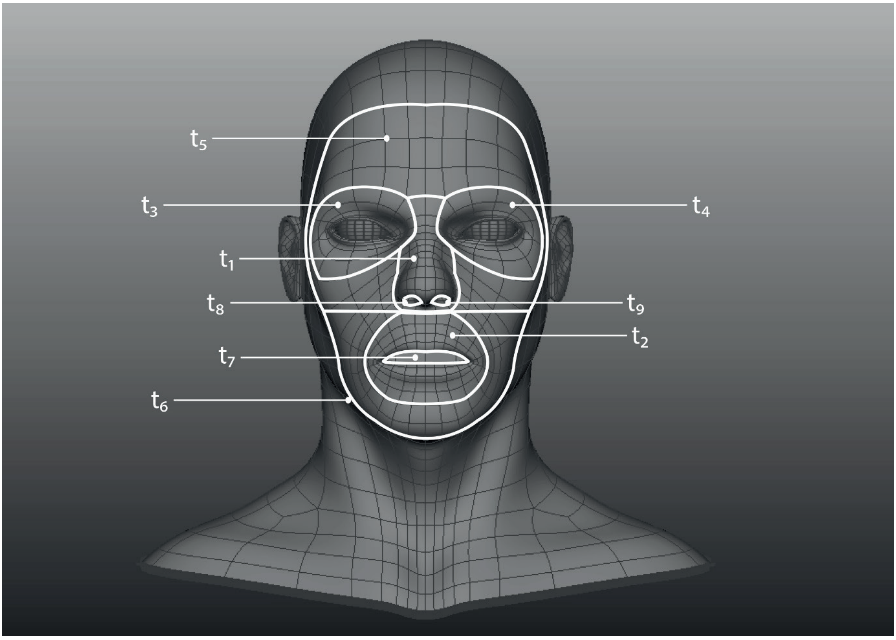

Fig. 2. A face with nine spatial regions indicated.

Moreover, we can say that those objects are *located in* other specific objects, i.e. the nose (ℓ_1) and the lips (ℓ_2), without overlapping with them:

$LI(t_8, t_1) \wedge LI(t_9, t_1) \wedge LI(t_7, t_2)$

But

$\neg P(\ell_8, \ell_1) \wedge \neg P(\ell_9, \ell_1) \wedge \neg P(\ell_7, \ell_2)$

We now move to the next section, in which we will discuss and clarify the impact modal operators make in our language. This will exhaust the whole language MDML, which, we hold, fulfils the desiderata we introduced in the first section.

5. *Modal operators*

We will now discuss and explain the behaviour of the modal components of MDML. The addition of modalities to mereotopology was inspired by Vakarelov (2008). This section proceeds in two steps: in step one, we informally explain why we wish to have modal operators in our language; we give informal interpretations of the operators and we provide a formal semantics for the whole language MDML. The formal structure we will use is that of neighbourhood semantics; in step two we further refine our face scenario to show how the modal operators impact the description of the human body.

5.1. *Adding modalities to the language*

Modalities deal with intensional contexts. An intensional context is a context in which the truth of a compound sentence can't be determined by the truth of its compounds. For example, knowing that it is raining outside (i.e. that the sentence 'it is raining outside' has truth value true), doesn't tell us anything about the sentence 'it is necessary that it is raining outside'. This is because the modality 'it is necessary' generates an intensional context. To deal with such problem, it is common, in modal logic, to assume that there is a system which comprises the many possible scenarios that can occur. Saying that it is necessary that it is raining outside amount to say that in each possible scenario it is raining outside. This can be extended easily to predicates: saying that it is necessary that some entity a has the property R amount to say that in each possible scenario, the entity a has the property R. This approach permits an essentialist interpretation of the modal operator $\Box$: if we interpret 'being an essential property' as standing for 'possessing that property in every possible scenario', then $\Box R(a)$ means, by definition, that a has the essential property of being R. In this paper, we will follow such an interpretation of the box-modality.[29]

We now provide the formal semantics for the box-modality: this semantical structure will be the one that provides an interpretation to our whole theory MDML. The model we will give is based on neighbourhood frames: choosing neighbourhood semantics over standard Kripke semantics will allow the interpretation of the language to be context-sensitive (i.e. it will permit to distinguish different ways we can design the possible scenarios with a maximal amount of freedom).

We define a neighbourhood frame as:

(D. 14) A quadruple (W, Obj, $\wp U$, N) where W is a non-empty set of possible states of affair, Obj a non-empty set of individuals, $\wp U$ a non-empty set of spatial regions[30] and N is a neighbourhood function, i.e. $N{:}W \longmapsto \wp(\wp(W))$.

[29] We will not enter the debate concerning essentialism, we refer for a thorough discussion about mereological essentialism to Casati & Varzi (1999).

[30] Obj and $\wp U$ might be countably infinite and are usually called universes of discourse.

Informally, a neighbourhood function is a function that associates with each state of the system all the formulas that are necessary in that particular state; note that formulas are seen as sets of states (i.e. $\wp(W)$), in particular, each formula is associated with the set of states in which the formula is true. This allows for the possibility of specifying different necessary formulas in different states, without having to rely on the structure of the whole system and on the logical relations between formulas.

From a modal neighbourhood frame we can move to modal neighbourhood models for our language:

(D. 15) Let $\mathfrak{F}=(W, Obj, \wp U, N)$ be a modal neighbourhood frame. A model based on $\mathfrak{F}$ is a tuple $(W, N, Obj, \wp U, \sigma, \tau)$, where σ is an individual function defined on both X^{Obj} (the set of objects variables) and X^{U} (the set of cell variables) and τ is a relation function defined on $\mathscr{P}\times W$ (where $\mathscr{P}=\{P;C\}$, i.e. the set whose elements can only be P or C). $V=(\sigma, \tau)$ is called a $\mathfrak{F}$-assignment if

i. For all x^{U}, $\sigma(x^{U}) \in \wp U$ and for all x^{Obj}, $\sigma(x^{\mathrm{Obj}}) \in Obj$.
ii. For $P \in \mathscr{P}$ and $w \in W$, $\tau(P,w)$ is a binary relation on Obj or $\wp U$, i.e. $\tau(P,w) \subseteq Obj\times Obj$ or $\tau(P,w) \subseteq \wp U\times\wp U$; for $C \in \mathscr{P}$ and $w \in W$, $\tau(C,w)$ is a binary relation on U, i.e. $\tau(C,w) \subseteq \wp U\times\wp U$.

We call $\mathfrak{M}=(\mathfrak{F},V)$ a modal neighbourhood model. The two functions σ and τ depend for their assignment on the interpretation I we gave in sections 3.2 and 4.1, i.e. they should always assign elements of the respective domains according to the adjacency spaces defined in section 3.2.

Before giving the truth definition for formulas in our language, we need to introduce the concepts of *constant designation* and *variant assignment*:

(D. 16) A constant designation is a function ρ that assigns a member of the universes of discourse to each constant symbol in the language in relation to a state $w \in W$:

i. $\rho(t,w) \in \wp U$;
ii. $\rho(\boldsymbol{\ell},w) \in Obj$.[31]

(D. 17) Let $\mathfrak{F}$ be a modal neighbourhood frame and V a $\mathfrak{F}$-assignment. A x-variant of a variable assignment σ, i.e. either $\sigma(t/x^{\mathrm{U}})$ or $\sigma(\boldsymbol{\ell}/x^{\mathrm{Obj}})$, is an assignment just like σ except perhaps for the members of either $\wp U$ or Obj that are assigned to x. Formally, for any variable x and for any constant $t \in$

[31] We are assuming that it is always possible to assign a member of the universes of discourse to the constants, i.e. $\exists u_i^{Obj}(u_i^{Obj} \in Obj \wedge \rho(\boldsymbol{\ell},w)=u_i^{Obj})$ and $\exists u_i^{\mathrm{U}}(u_i^{\mathrm{U}} \in \mathrm{U} \wedge \rho(t,w)=u_i^{\mathrm{U}})$. Moreover, we assume that the constant designation is rigid: this amount to say that $\forall w_i \forall w_j((w_i \in \mathrm{W} \wedge w_j \in \mathrm{W}) \rightarrow (\rho(\boldsymbol{\ell},w_i)=\rho(\boldsymbol{\ell},w_j)))$ and $\forall w_i \forall w_j((w_i \in \mathrm{W} \wedge w_j \in \mathrm{W}) \rightarrow (\rho(t,w_i)=\rho(t,w_j)))$.

$\wp U$ or $\ell \in Obj$, we can define two distinct new assignments depending on whether the constant is t or ℓ:

The first is $V(t/x^{\mathrm{U}}) = \langle \sigma(t/x^{\mathrm{U}}), \tau \rangle$, where:

$$\sigma(t/x^{\mathrm{U}})(y^{\mathrm{U}}) = \begin{cases} \sigma(y^{\mathrm{U}}) & y^{\mathrm{U}} \neq x^{\mathrm{U}} \\ t \quad (y^{\mathrm{U}}) & y^{\mathrm{U}} \neq x^{\mathrm{U}} \end{cases}$$

The second is $V(\ell/x^{\mathrm{Obj}}) = \langle \sigma(\ell/x^{\mathrm{Obj}}), \tau \rangle$, where:

$$\sigma(\ell/x^{\mathrm{Obj}})(y^{\mathrm{Obj}}) = \begin{cases} \sigma(y^{\mathrm{Obj}}) & y^{\mathrm{Obj}} \neq x^{\mathrm{Obj}} \\ \ell \quad (y^{\mathrm{Obj}}) & y^{\mathrm{Obj}} \neq x^{\mathrm{Obj}} \end{cases}$$

We finally define truth in the model for the formulas of our language MDML:

(D. 18) Let M be a modal neighbourhood model. Then the truth definition for formulas of MDML is given inductively as follows:
a) $V(P(t_1, t_2),w)=1$ iff $(\rho(t_1),\rho(t_2)) \in \tau(P,w)$;
b) $V(P(\ell_1, \ell_2),w)=1$ iff $(\rho(\ell_1),\rho(\ell_2)) \in \tau(P,w)$;
c) $V(C(t_1, t_2),w)=1$ iff $(\rho(t_1),\rho(t_2)) \in \tau(C,w)$;
d) $V(P(x^{\mathrm{Obj},} y^{\mathrm{Obj}}),w)=1$ iff $(\sigma(x^{\mathrm{Obj}}),\sigma(y^{\mathrm{Obj}})) \in \tau(P,w)$;
e) $V(P(x^{\mathrm{U}}, y^{\mathrm{U}}),w)=1$ iff $(\sigma(x^{\mathrm{U}}),\sigma(y^{\mathrm{U}})) \in \tau(P,w)$;
f) $V(C(x^{\mathrm{U}}, y^{\mathrm{U}}),w)=1$ iff $(\sigma(x^{\mathrm{U}}),\sigma(y^{\mathrm{U}})) \in \tau\ (C,w)$;[32]
g) $V(\neg\alpha,w)=1$ iff $V(\alpha,w)=0$;[33]
h) $V(\alpha \wedge \beta,w)=1$ iff both $V(\alpha,w)=1$ and $V(\beta,w)=1$;
i) $V(\Box\alpha,w)=1$ iff $\mathrm{S} \in \mathrm{N(w)}$, where the set S is the set containing all states s s.t. $V(\alpha,s)=1$;
j) $V(\forall x(\alpha),w)=1$ iff either $\sigma\ (t/x^{\mathrm{U}})(\alpha\ (x^{\mathrm{U}}),w)=1$ for all x-variants $\sigma(t/x^{\mathrm{U}})$ of σ or $\sigma(t/x^{\mathrm{Obj}})\ (\alpha(x^{\mathrm{Obj}}),w)=1$ for all x-variants $\sigma\ (t/x^{\mathrm{Obj}})$ of σ.[34]

This exhaust our modal neighbourhood semantics for MDML.

[32] The conditions (a) and (d), (b) and (e), and (c) and (f) differ only in the type of function that is applied to either the constants or the variables. This is related to the fact that our interpretation is a static one, where the constants maintain their interpretation through the different situations on which the formulas are evaluated, while the variables can change meaning depending on the situation.

[33] In order to obtain the conditions for $V(\alpha,w)=0$ for all α of the form (a)-(f), the symbol $\in$ in conditions (a)-(f) should be substituted with symbol $\notin$.

[34] As far as we have two domains that contain distinct kinds of entities, the disjunction is exclusive.

5.2. *The final scenario*

We provide a final scenario for the application of the whole language MDML to a face. For the scenario, we will use the face of (fig. 2) and face of (fig. 3). We will imagine two distinct possibilities for the way scenarios are designed: the first possibility is one in which the modelled face should be employed for a horror movie with zombies; the second possibility is one in which the modelled face has to be employed for an anatomical medical research; moreover, we make a second example where we show the expressive power modalities have when dealing with the difference between local and general information about the structure of the body.

Imagine we have the face of (fig. 3) on which we identified all the following information: ℓ_1: the nose; ℓ_2: the lips; ℓ_3: the right eye; ℓ_4: the left eye; ℓ_1: the upper part of the head; ℓ_6: the whole face; ℓ_7: the mouth's hole; ℓ_8: the right naris and ℓ_9: he left naris. Moreover, as information, we have the regions that are associated to the objects above reported. Imagine, further, that we are trying to describe the face of a zombie for a horror movie: this zombie might or might not have eyes, but if he has both, then the left eye may strain close to its mouth. We will call this situation *the zombie situation*. A second situation we will describe and analyse involves a nose which is positioned on the arm. We call this situation *the nose-on-arm situation*.

In the zombie situation, since the eyes might be present or absent, the fact that the eyes are part of the head is not essential. However, if the zombie does have both eyes, we allowed for the possibility that the left eye is straining towards the mouth, putting the former in contact with the latter. We can describe this situation with MDML in the following way:

$$(P(\ell_3, \ell_6) \wedge P(\ell_4, \ell_6)) \wedge (\neg P(\ell_3,\ell_6) \wedge P(\ell_4, \ell_6)) \wedge (P(\ell_3, \ell_6) \wedge \neg P(\ell_4, \ell_6)) \wedge (\neg P(\ell_3,\ell_6) \wedge \neg P(\ell_4, \ell_6))$$

The latter formula says that the zombie might have both eyes, only one of the two eyes or none of them. We can also add the following formula:

$$\Diamond\, (P(\ell_3, \ell_6) \wedge P(\ell_4, \ell_6)) \rightarrow C(\ell_4, \ell_2)$$

This formula describes the fact that it is possible that, if the zombie has both eyes, then the left eye is in contact with the lips (as shown in Fig. 3). If we changed the modal operator with the box-operator, we could have described a different situation in which the left eye *must be straining* towards the lips.

In the nose-on-arm situation, we can see a different use we can make of the modal operators. Imagine that we want to describe the fact that it is essential for the nose to be connected to the larynx to allow a correct respiration action in a human being. But, for aesthetical purposes the nose can

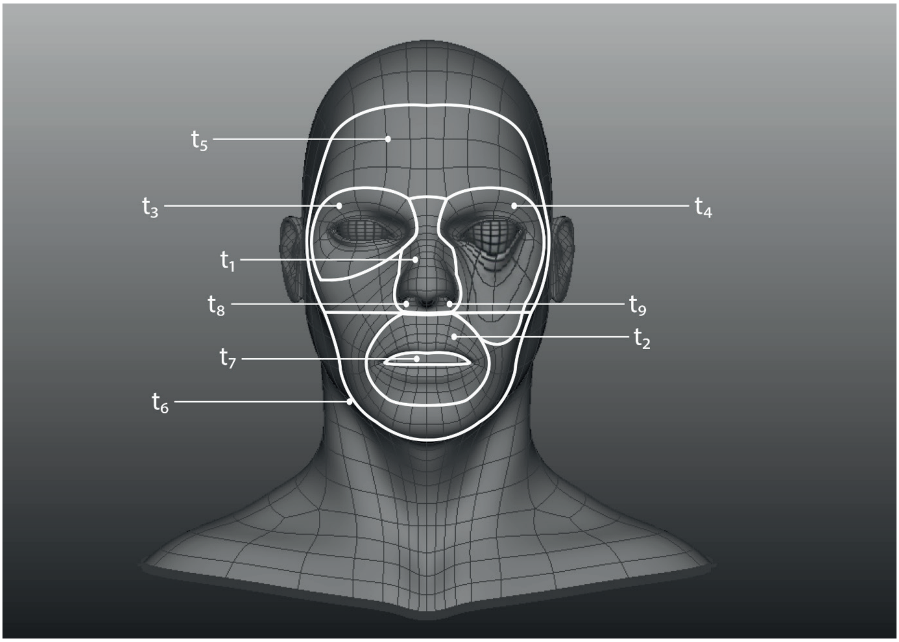

Fig. 3. A face with nine spatial regions indicated, where the left eye has strained close do the mouth.

also be placed on the arm, disconnected from the larynx and the rest of the respiratory system. Moreover, consider a further situation in which someone wants to argue that it is in fact possible that human beings evolved in different manners and that it isn't necessary at all for the nose to be connected to the larynx to have correct respiration, but this is only a contingent fact. MDML has the expressive power to describe and distinguish all those different situations.

In the first scenario, we have a doctor who wants to describe the fact that the nose is essentially connected to the larynx to function properly.[35] If the larynx is an object denoted by the constant symbol t_{10}, whose spatial region is t_{10}, we can express the basic idea behind our scenario as follows:

$\Box(C(t_1, t_{10}))$

Obviously this formula is true in our interpretation only if $V(C(t_1, t_{10}), w) \in N(w)$; in order to have the latter, it must be the case that the subset of all the possible

[35] See Krohs & Kroes (2009) and Houkes & Vermaas (2010) for discussions on functions in biological systems and artefacts.

scenarios (i.e. W) in which $C(t_1, t_{10})$ is true is a member of the set of subsets that N associates with the particular scenario w (i.e. the situation that is being modelled). Therefore, the modeller can modify the behaviour of the function N to obtain the above situation, achieving the goal of making the connection between nose and larynx essential. However, an artist that judges the position of the nose from an aesthetic point-of-view might not want that connection to be essential. In this case, the artist would only need to evaluate the formula in a different scenario $s \in W$, in which the subset associated with $C(t_1, t_{10})$ is not a member of the set of subsets associated with that scenario s. This amount to say that at s:

$$\neg\Box(C(t_1, t_{10}))$$

which can be redefined as:

$$\Diamond \neg(C(t_1, t_{10}))$$

Finally, to show how powerful our language is, imagine the last scenario in which a philosopher wants to argue, against the doctor, that the fact that the nose has to be connected to the larynx to function properly is only a contingent fact. In this case, the philosopher, rather than changing the scenario in which he evaluates the formula $\Box C(t_1, t_{10})$, can change the behaviour of the neighbourhood function, so that N associates a different set of subsets to the scenario w.

Those scenarios manage to show how the addition of the modal operators allows for the description of different possible scenarios related to the necessity or the possibility of different things happening. Moreover, having a neighbourhood semantics allows the modeller to specify the set of modal formulas that he reckons as necessary in the scenario he is describing, allowing different descriptions of the same scenario. Furthermore, neighbourhood semantics give the chance to indicate some necessary conditional formulas that can be employed later by an automatic deducing system to gather extra information from the one manually provided by the modeller himself.

In the next and final section, we describe a possible application of our language MDML.

6. *MDML and computer graphic grids: a possible application*

We finally add the formal tools of the mereotopological theory we introduced to mathematical objects (i.e. meshes) we obtain in computer graphics when we reproduce the human body.

The technique we will be referring to is that of big database interpolation (BDI). BDI is a well-established computer graphic technique used to reproduce the structure of a human body. We will focus on the first step of BDI, i.e. constructing the input data. The basic idea is that MDML can increase the quality and the precision of the input data. This increased quality of the input will then help during the second step, i.e. when the input is used to query the databases to obtain the computational reproduction of a human body.

Normally, when BDI is applied, the initial data comes from a scan of a real surface, e.g. a face or the whole body. This scan is then translated into a point cloud. A point cloud, as the term suggests, is a series of points placed on a matrix. Those matrices are usually in seven dimensions, where three of those are the spatial dimensions (height, length and depth) and the remaining four are the three primitive colour parameters and the opacity information.[36] This seven-dimension matrix can be reduced to a three-dimensional one, in which the colour and opacity information is ignored, keeping only the spatial information. A scan then becomes a cloud of spatially identified points. This process allows a computer to partially deal with the information that comes from a high-definition scan of a real object, but, at this level, there is still a lack of topological information about the relations that occur between those points. To obtain such additional information, one of the possibilities is to construct a mesh (grid) that corresponds to a tessellation of the geometrical surface that we wish to reproduce. In computer graphics, the way meshes are obtained is through the indication of which points are connected by edges. Those edges, in turn, form closed surfaces that constitute polygons. The basic polygon that we will consider in this paper is the square.

Those two practices – transforming a scan into a point cloud and then constructing the mesh on the point cloud – are commonly performed by algorithms.[37] In addition, there are also further algorithms, e.g. Catmull-Clark's algorithm (1978), that allow for an increase in the quality of the mesh already constructed. With a refined mesh in hand, BDI techniques then automatically proceed to query databases, searching for images that correspond to the mesh just constructed. However, this search carries with it a high computational complexity, making the whole BDI technique quite expensive in terms of computational resources. This is what justifies an approach that tries to reduce the resources required to query the databases. MDML can help improving the quality of the mesh, therefore reducing such resource requirements. We now show how our theory can be applied to the input.

[36] The colour and opacity information is commonly referred to as RGBA information. R stands for red, G stands for green and B stands for blue, while A is the parameter referring to the opacity.

[37] For a thorough discussion about the mathematization of geometrical surfaces see Sack and Urrutia 1999.

Ordinarily, meshes carry information only about the relations that occur between points, but no information about the relations among the polygons that are formed through the mesh generation process. MDML can provide such additional information about the relations among those polygons. In this paper, we will limit ourselves just to squares. The reason for doing so is that, as we previously saw in this section, squares are the standard polygons that compose meshes when automatic algorithms are used. It is important to note that our approach requires that the additional information that comes from MDML must be provided manually by a graphic designer or, in general, by a user the first time the theory is applied to a specific case. The advantage is that once the whole information (i.e. the whole structure of relations) is provided, in successive steps, only some of the relations can be given by the user and automatic procedures can be developed to complete the whole structure.

The core idea of our approach is to interpret particular parts of the human body, which we will call *extended landmarks*,[38] as regions on an adjacency space. Those regions, as we previously said, are formed by discrete cells, that, in the specific case, are the squares that the automatic mesh generation algorithms have created on the point cloud. It is then possible to introduce all the relations we saw in section two, such as the relation of part, connection and location and, moreover, it is possible to specify which relations are essential given a certain context. Those relations, defined on regions, aid in providing the additional information about the mesh we needed to improve the quality of the input data. A small practical example might help understanding the procedure. Let's suppose we are trying to reproduce a face. To have a digital reproduction of the face that can both be modelled and be computationally modified, we initially must scan the real-world face. On the face's scan, we then run a tessellation algorithm which will provide a mesh, made of squares, that corresponds to the two-dimensional shape of the face. On this mesh, we can choose some regions that will become our extended-landmarks. Those choices are usually aided by studies in anatomical anthropology, that indicate which parts of a face are fundamental (see for example Vezzetti and Marcolin (2012)). At this point we have all we need to apply MDML. Suppose we use the final face example of fig. 2. For such a face, we had the following extended landmarks:

EXTENDED LANDMARKS: t_1: the nose; t_2: the lips; t_3: the right eye; t_4: the left eye; t_5: the upper part of the head and t_6: the whole face.

[38] We do so because with the term landmark, in HRs reproduction, it is meant a peculiar point in the point cloud. Since we are dealing with extended spatial regions, instead of points, we add the adjective 'extended' to the term.

SPECIAL OBJECTS: ℓ_7: the mouth's hole; ℓ_8: the right naris and ℓ_9: he left naris.

At that point, the graphic designer (or the computer scientist) will indicate which regions are connected. In our case:

$C(t_1, t_2) \wedge C(t_1, t_3) \wedge C(t_1, t_4)$[39]

But

$DC(t_2,t_3) \wedge DC(t_2, t_4) \wedge DC(t_3, t_4)$

Moreover:

$PP(t_1, t_5) \wedge PP(t_3, t_5) \wedge PP(t_4, t_5) \wedge PP(t_5, t_6)$

But

$\mathrm{DR}(t_2, t_5)$

As for the special objects:

$LI(t_8, t_1) \wedge LI\ (t_9,t_1) \wedge LI\ (t_7,t_2)$

But

$\neg P(\ell_8,\ell_1) \wedge \neg P(\ell_9, \ell_1) \wedge \neg P(\ell_7, \ell_2)$

Finally, assuming the context in which the face is modelled is that of an anatomical medical research, the designer can also indicate some essential relation between extended landmarks or between the special objects:

$\Box(\mathrm{LI}\ (t_8, t_1) \wedge \mathrm{LI}\ (t_9, t_1))$

Even though it might seem laborious to provide all the initial relations, this step must be done only once. The trick is to provide names to the extended-landmarks, so that during successive modelling processes, the graphic designer will only need to specify those names on the mesh and the computer will then generate the relational structure by itself.

The added information that comes from MDML greatly enhances the quality of the input data, which, in turn, increases the quality of the output image that has been sought in the database.

[39] We use here the connection predicate to keep the example simple. Obviously, the modeler can make use of stronger relations, such as that of tangential part or external connection.

7. *Conclusion*

We will now assess MDML, showing how it stands with respect to the four desiderata introduced in the first section of this paper.

According to desideratum 1, a formal language designed to describe the human body has to be able to give both a micro- and a macro-description of the human body. Our language achieves this by employing the full strength of a first-order logical language as a starting base: since the domains we are working with contain objects and spatial regions, without any reference to their scale, it doesn't make any difference if the description is at a micro or macro level.

According to desideratum 2, a formal language designed to describe the human body has to be fine grained enough to describe all the parts of the body and their relationships. Again, we fulfil the desideratum by employing a first-order logical language. Having two domains for objects and spatial regions, makes it possible to referencing to all the objects and spatial regions that are present in a human body; moreover, having a countable number of predicate sets allows for a description of all the possible relations occurring between those parts.

According to desideratum 3, a formal language designed to describe the human body has to be able to capture essential features of the structure of the body. Our language fulfils this desideratum by having a modal component, where our modality is interpreted as 'being an essential property'.

According to desideratum 4, a formal language designed to describe the human body for computer graphics applications has to be able to distinguish different contexts of description. Our language fulfils this desideratum by interpreting the modalities through a neighbourhood semantics; this allows for great versatility for the modeller, who can change the essential features of his description depending on the scenario he is describing.

Concluding, we started with the problem of having a language expressive enough to give a rigorous description of the human body. We proposed a formal language (MDML) which we believe is expressive enough to capture many features of the human body, therefore solving the problem we posed. We then showed how each component of MDML impacts the expressivity of it and how the quality of the description of a face increases when we add different pieces of the language. We finally proposed a possible application of our language to computer graphics, increasing the quality of a mathematical technique which is among the most promising in the field.

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References

Bastioni, Manuel, Pierluigi Graziani, 2011, "Quanta Matematica serve per costruire un essere umano?", in *Lettera Matematica Pristem*, 78: 26-37.

Calosi, Claudio, Pierluigi Graziani, 2014, eds., *Mereology and the Sciences. Parts and Wholes in the Contemporary Scientific Context*, Springer, Berlin.

Casati, Roberto, Achille C. Varzi, 1999, *Parts and Places: The Structures of Spatial Representation*, MIT Press, Cambridge MA.

Catmull, Edwin, Jim Clark, 1978, "Recursively generated B-spline surfaces on arbitrary topological meshes", in *Computer-Aided Design*, 10, 6: 350-355.

Chellas, Brian F., 1980, *Modal Logic*, Cambridge University Press, Cambridge.

Donnelly, Maureen, 2003, "Layered Mereotopology" in G. Gottlob and T. Walsh, eds., *Proceedings of the 18th International Joint Conference on Artificial Intelligence* (IJCAI-03), Morgan Kaufmann Publishers Inc., San Francisco CA, USA: 1269-1274.

Donnelly, Maureen, 2004, "On Parts and Holes--The Spatial Structure of the Human Body", in Marius Fieschi, Enrico Coiera, and Yu-Chan Jack Li, eds., *Proceedings of the 11th World Congress on Medical Informatics* (MedInfo-04): 351-356. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.90.7535&rep=rep1&type=pdf.

Galton, Antony, 2014, "Discrete Mereotopology", in Claudio Calosi, and Pierluigi Graziani, eds., *Mereology and the Sciences. Parts and Wholes in the Contemporary Scientific Context*, Springer, Berlin: 293-321.

Houkes,Wybo, Pieter E. Vermaas, 2010, *Technical Functions*, Springer, Berlin.

Jansen, Ludger, Stefan Schultz, 2014, "Crisp Islands in Vague Seas: Cases of Determinate Parthood Relations in Biological Objects2, in Claudio Calosi and Pierluigi Graziani, eds., *Mereology and the Sciences. Parts and Wholes in the Contemporary Scientific Context*, Springer, Berlin: 163-188.

Krohs, Ulrich, Peter Kroes, eds., 2009, *Functions in Biological and Artificial Worlds: Comparative Philosophical Perspectives*, The MIT Press, Cambridge (MA).

Montague, Richard, 1970, "Universal Grammar", in *Theoria* 36: 373-398.

Ottesen Johnny T., Mette S. Olufsen, Jasper K. Larsen, 2004, *Applied Mathematical Models in Human Physiology*, SIAM: Society for Industrial and Applied Mathematics.

Pacuit, Eric, 2014, “Neighbourhood Semantics for Modal Logic”, http://web.pacuit.org/files/nbhd-v3-CH1.pdf.

Sack Jörg-Rüdiger, Jorge Urrutia J., 1999, *Handbook of Computational Geometry*, Elsevier Science B. V., North Holland, Amsterdam.

Scott, Dana, 1970, “Advice in modal logic”, in Karel Lambert, ed., *Philosophical Problems in Logic: Some Recent Developments*, D. Reidel, Dordrecht: 143-173.

Vakarelov, Dimiter, 2008, “A modal approach to dynamic ontology: modal mereotopology”, in *Logic and Logical Philosophy*, 17, 1-2: 163-183.

Varzi, Achille, 2016, “Mereology”, in *The Stanford Encyclopedia of Philosophy*, Edward N. Zalta, ed., https://plato.stanford.edu/archives/win2016/entries/mereology/

Vezzetti, Enrico, Federica Marcolin, 2012, “3D human face description: landmarks measures and geometrical features”, in *Image and Vision Computing Archive*, 30, 10: 698-712.

Vitruvius, Marcus Pollio, 1914, *Ten Books on Architecture*, Eng. tr by Morris Hicky Morgan, Harvard University Press, Cambridge MA.

Zhuang-hu, Liu “Neighbourhood Semantics of Modal Predicate Logic”, viewed September 24, 2017, http://www.phil.pku.edu.cn/cllc/archive/papers/logic/LiuZhuanghu/Liuzhh_ModalPL.pdf.

Zinkovsky, Anatoly V., Victor A. Sholuha, Alexandre A. Ivanov, 1996, *Mathematical Modelling and Computer Simulation of Biomechanical Systems*, World scientific Publishing, Singapore.