Free will is compatible with randomness

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Abstract: It is frequently claimed that randomness conflicts with free will because, if our actions are the result of purely random events, we must lack control over them. The paper challenges this view. After arguing for a product rather than a process notion of randomness, it uses an intuitive two-stage, contextual definition of free choice to show that, relative to this definition, randomness is compatible with free will. But we also stress that the argument is relative in nature: the conclusion holds only if free will is itself metaphysically possible, a claim that is beyond the purview of the paper.

Keywords: Free will; randomness.

1. *Introduction*

The claim that randomness is incompatible with freedom is stated by many authors as an obvious fact (Doyle 2011). The following quotations are illustrative.

One horn of this dilemma is the argument that if an action was caused or necessitated, then it could not have been done freely, and hence the agent is not responsible for it. The other horn is the argument that if the action was not caused, then it is inexplicable and random, and thus it cannot be attributed to the agent, and hence, again, the agent cannot be responsible for it (Russell 1995: 14).

On the other hand, if indeterminism is true, then, though things could have happened otherwise, it is not the case that we could have chosen otherwise, since a merely random event is no kind of free choice. That some events occur causelessly, or are not subject to law, or only to probabilistic law, is not sufficient for those events to be free choices (McGinn 1993: 80).

* Partially supported by Marie Curie FP7-PEOPLE-2010-IRSES Grant RANPHYS.
** Partially supported by a Computer Science Summer Research Scholarship, 2013-2014.

philinq IV, 2-2016, pp. 37-52 Submitted: August 2015
ISSN (print) 2281-8618-ETS Accepted: June 2016

Yet if an uncaused action is a random happening, then this no more comports with human value than does determinism. Random acts and caused acts alike seem to leave us not as the valuable originators of action but as an arena, a place where things happen, whether through earlier causes or spontaneously. Clearly if our actions were random, like the time of radioactive decay of uranium 238 emitting an alpha particle, their being thus undetermined would be insufficient to ground human value or provide a basis for responsibility and punishment (Nozick 1981: 91-92).

Our aim is to challenge this view: we will argue that randomness in an agent's choices is independent of whether she is free to make those choices. The allegedly intimate relation between free will and (absence of) randomness has been noticed and examined from the very beginning of the debate about free will. Despite the important role it plays in this debate, the notion of randomness itself is often left unexplicated, or is identified with chance or even indeterminism (Doyle 2011). Here is a quote similar to the previous ones, this time using chance rather than randomness. According to Ayer:

Either it is an accident that I choose to act as I do or it is not. If it is an accident, then it is merely a matter of chance that I did not choose otherwise; and if it is merely a matter of chance that I did not choose otherwise, it is surely irrational to hold me morally responsible for choosing as I did (Ayer 1954: 285).

But what is randomness? The above passages may suggest that it has something to do with the way outcomes are produced – a process conception of randomness on which random actions are chancy actions (Eagle 2005). But this is not the only or even the most intuitive way to understand the notion of randomness. There is also a product conception of randomness (Eagle 2014) on which random happenings are haphazard, lacking any discernible or explicable pattern. This conception is the object of deep (and continuing) mathematical study (see Calude 2002; Downey and Hirschfeldt 2010), but it is also, so we argue, the notion of randomness that is of most theoretical and philosophical interest if we are to understand the relation between free will, indeterminism, and randomness.

The plan of the paper is the following. We first describe what we take to be a fruitful framework for the discussion of free will and we propose a simple, two-stage, contextual (not absolute) definition of free will based on this framework. We operate with this definition throughout the paper, but we do not claim that it is the only or even best way to analyse free will. It does, however, comport with much of what theorists have wanted to say about free will, and in particular where it intersects with the idea of randomness. We then examine the notions of indeterminism and randomness and their mutual relations from within this framework. We next show that neither of them implies the other.

Finally, we provide a detailed analysis showing that randomness is not incompatible with free will.

Note that our arguments are doubly relative. Not only do they not rule out other conceptions of free will (indeed, we think that progress in this area depends on rigorously developing competing frameworks). They also do not provide a proof of the metaphysical possibility of free will thus conceived, but only of various combinatorial possibilities should free will indeed be possible.

2. *Agents, objects, contexts and choices*

We start by defining four main components of a manifestation of free will: agent, objects, contexts and choices. An agent is an entity that can make a choice (decision) in some context. By choice we mean that the agent is able to pick an element from an abstract set of objects. The context C gives the environment and the relevant constraints for the choice to be made. A context may include the position in space and time where an agent must make a choice, as well as various constraints the objects should satisfy. The object that the agent picks is called the chosen object in the context C.

We illustrate these definitions with some examples. Consider first a girl at a pet store looking to buy a pet. Here the agent can be taken to be the girl, the objects are the pets available for purchase in the shop, and the context is the shop at some time; the girl can choose any pet. On the other hand, an individual animal in the shop can be an agent in the same context: the objects are the foods the animal can eat and a choice consists in selecting one of the available foods.

In the same context, the same animal can be an agent for the set of actions {eat, not eat}, where the choice consists in deciding on an action in that set. Considered as the schema for a broad notion of freedom, this characterisation is extremely abstract. Perhaps it does not even require agents to be conscious.[1] Of course, where free will is the issue more constraints are needed.

[1] An extreme (and contentious) case is the following. Consider as agent an experimental physicist and as objects the set of experiments she can perform; the context includes the physical set-up for the experiment at a given location and time. For example, a possible choice is to look through a microscope for a particle in an uncertain location. Now consider a particle with an uncertain location and an uncertain velocity in a given context. When one looks at the particle through a microscope and locates it, the particle gives an answer *here I am*: a given location from the available ones in that context was chosen by the particle. Cf. quantum physicist Zeilinger (2006), who calls these "the two freedoms: first the freedom of the experimenter in choosing the measuring equipment – that depends on [his] freedom of will; and then the freedom of nature in giving him the answer it pleases. The one freedom conditions the other…".

To see what these might be, note first that the idea of context is crucially related to the idea of possibility. The choices the agent can make are assumed to be contextually possible, i.e. possible in the given context. They may or may not be considered rational, morally justified or politically acceptable. Contextual possibility should also be distinguished from other notions of possibility; for instance, an agent may have the ability to scuba dive, so that it is possible for the agent, given her abilities, to scuba dive, even though it is not possible in a given context because the agent is not close to water. As it stands, contextual possibility is also different from a kind of intentional contextual possibility in which the agent has sufficient control over her ability to deliberately select certain outcomes (an ability Alfred Mele calls intentional; see 2006: 17-19). Intentional contextual possibility implies contextual possibility, but the converse does not hold. For example, an agent can intentionally flip a coin, but while it is possible in the context for her to get heads she cannot intentionally get heads. It then also follows that she cannot freely choose to get heads. To understand why, we need to address the constraints needed in an account of free choice.

3. *A definition of free will*

In the vast literature on free will (see Doyle 2011) a lot of misunderstanding comes from the fact that free will has been understood in numerous different ways. In what follows we try to avoid confusion by working with a simple, two-stage, contextual (not absolute) definition of free will. We stress again that we do not claim that this proposal is the uniquely right definition: its goal is mainly to propose a more precise and detailed framework for studying the relation between free will, determinism, and randomness.

The definition of free will models the idea that an agent has free will in some context if it has the ability to make a decision that is not completely determined by or the result of prior events. More precisely, an agent A acted freely in the context C with respect to the set of objects $O_C(A)$ if A could have acted differently to the way it did and A had full control over the outcome of the choice. This means that in the context C:

- (P) The set of objects $O_C(A)$ available to A contains at least two elements and every choice for A is possible in the given context.
- (C) The agent A has full control over which of the objects in $O_C(A)$ to choose.

This is a two-stage Valerian definition.[2] The main point of the Possibility Assumption, (P), is that in context C there are at least two objects in $O_C(A)$ available to *A* to choose from. (P) thereby guarantees that the act of choice is meaningful and can lead to different outcomes, something that is impossible if $O_C(A)$ has fewer than two elements. Two further points: $O_C(A)$ depends on the context C and can vary with C – by changing the context C some elements may disappear and new elements can be added to $O_C(A)$. Note also that the identity of $O_C(A)$ may not be completely known by *A*: that is, *A* may not exhaustively know what, in the context, is possible for her. The Choice Assumption, (C), supplements (P) by claiming that the agent has full control over which option to choose.[3] (C) thereby suggests that in the case of free choice the contextual possibility of a choice is intentional contextual possibility, and so involves intentional ability in Mele's sense. These two assumptions satisfy a broadly libertarian view of free will, i.e. to be free an agent must have the ability to do otherwise from what she did (McKenna 2009; for a detailed discussion regarding the Possibility Assumption see Timpe 2008: ch. 3 and 4).[4]

We stress that the contextual availability of every element in $O_C(A)$ is essential to free choice. For example, if *A* is a human being, then the set $O_C(A)$ = {to be immortal, not to be immortal} violates the condition of contextual possibility: "to be immortal" is not available to *A* if *A* is human (although it

[2] Dennett (1978: 293) called his model of free will "Valerian" after Paul Valéry who was quoted as saying that it takes two to invent anything. The one makes up combinations; the other one chooses.

[3] The idea of full control may seem contentious, since it suggests i) that such an agent has powers that no ordinary agent has (for example, the power to generate and arrange at will the mental states involved in evaluating the various options faced by the agent) and ii) that the options open to the agents must be fully transparent to her, something we denied. We say nothing about i) here, apart from to note that 'full' is rather like 'perfectly' in 'perfectly flat': there are bound to be inbuilt constraints on what agents can do that do not, in the relevant sense, make for less than full control, just as the nanoscale ridges and bumps that engineers cannot avoid when constructing flat surfaces do not, in the relevant sense, make for less than perfect flatness. As for ii), we require only that the agent is aware of at least two members of $O_C(A)$, and that once she is aware of a member of $O_C(A)$ she has the intentional ability to select it.

[4] It may seem that (P) is only appropriate if the kind of control mentioned under (C) is what is sometimes called regulative control, a control involving the agent's ability to choose and act differently in the actual circumstances. Fischer (1994) and others have argued that this kind of control is not required for an agent to be morally responsible. An agent may be morally responsible for having done *x*, and to that extent have acted freely, if the agent had guidance control: a control involving the agent's responsiveness to reasons. (Here Fischer was persuaded by a form of argument first put forward by Frankfurt (1969).) Although we have said nothing about moral responsibility in this paper, we doubt that "freedom" without alternative possible courses of action is worth having. (It sounds for all the world like the freedom of a prisoner who is told he is free to leave but where things are arranged so that were he ever to get to the point of leaving the gates would close.) By contrast, the kind of account proposed here directly models the idea of "could have done otherwise" – a core component of the standard conception of free action.

may be available to certain mythical heroes). So *A* cannot act freely in such a context C. Below we briefly discuss the implication posed by such narrowed choice sets for the debate about free will and determinism, before turning to the challenge posed to free will by indeterminism and its association with randomness.

To sum up, the proposed definition of free will is contextual, not absolute nor global. It is in agreement with the conceptions of free will advanced by many other authors, including the quantum physicists' notion of free will (Laloë 2012); it does not assume any strong forms of rationality or moral responsibility. In addition, the definition satisfies the conditions of van Inwagen's classic (if contentious)[5] argument for incompatibilism (van Inwagen 1983: 70) and is supported by most of Dennett's (1978: ch. 15) reasons in favour of his two-stage model.

In what follows "free will" will always refer to the above definition. If the context C is understood we may write $O(A)$ instead of $O_C(A)$.

4. *Determinism, indeterminism, and randomness*

Is free will compatible with determinism? That is, can free will exist in a deterministic universe? While this is a crucial point of debate among those writing on free will, it is not central to our discussion. Broadly, determinism can be taken to be the view that given the state of the universe at time *t*, the way things go thereafter is fixed as a matter of natural law (Hoefer 2010). There are different variations on such an account, and many philosophers hold (at least some of) them to be compatible with the kind of free will we are discussing. We will not analyse these claims here, but point out that in general, supporters of such forms of compatibilism are left with the task of explaining how (P) holds in a universe in which the event of an agent's making a certain choice is fixed, for any earlier time *t*, by the state of the universe at *t*, including times *t* that preceded the existence of the agent. It is not clear that the set of objects available for an agent to choose from contains more than one element in that case (though the agent would presumably be unaware that there was only one object available for choice).[6] This problem

[5] One reason for the contentiousness of the argument has been the complaint that it trades on the wrong account of free will.

[6] There may be no such problem for those who reject (P) as a constraint on the notion of free will, and understand control to be guidance rather than regulative control (cf. footnote 4). There may also be no such problem if the relevant notion of what it is possible to do is understood to encompass more than just the physically possible worlds, since worlds with altered physical laws might permit the

is used by some to motivate a move to indeterminism. Here however, the alleged problem of randomness turns up: if indeterministic choice is random, how can we still say that an agent has free will?

Earlier we said that there are two distinct conceptions of randomness: a process and a product conception. We also suggested that the focus should be on the latter. The idea of a random event as the outcome of a chancy process faces a number of problems. First, is the notion of objective chance (understood as single-case objective probability) even coherent? Many have denied this. In an early reflection of such scepticism, the 5th BCE philosopher Leucippus wrote (see Doyle 2011: 133) that

> Nothing occurs by chance, but there is a reason and necessity in everything

Similarly, under the influence of mathematician A. de Moivre, Hume (1978: 56) called chance a mere word:

> […] there be no such thing as Chance in the world.

Moreover, the notion of process randomness has no mathematical formalization (and it is difficult to foresee an adequate solution due to the importance of external, e.g. physical, factors), and can be validated only through a theory (difficult, if not impossible, to test) or through an analysis of product randomness. The process notion has received a degree of rehabilitation because of the role some see it as playing in quantum mechanics, but for the reasons cited resistance continues to be strong.

Secondly, even if the notion is coherent it is not clear that it presents a radically different kind of challenge to free will from the challenge of determinism. In both cases, the choosing of a certain element from $O(A)$ is construed as the objective outcome of events that seem beyond the control of the agent. (This is precisely the point of the second quote of Section 1.) It is hard to see the move to chance as a game-changer.[7]

Authors like Ayer probably use the notion of chance in a more intuitive way, as suggested by the use of words like 'inexplicable' and 'unpredictable'. He would probably have agreed that there is no place for objective chance as an intrinsic feature of the world, and that the notion of randomness and chanciness should be operationalised in some way. In that case, randomness is better thought of as a product notion, a feature of the outcomes of certain

selection of a member of $O(A)$ whose actual selection is ruled out, given actual physical laws.

[7] See also Mele's discussion (in 2006) of the problem of luck in an indeterministic world (in his myth of Diana).

processes.[8] As observed in (Longo and Montévil 2015; see also Calude and Longo 2016),

> …randomness […] is in the interface between our theoretical descriptions and 'reality' as accessed by measurement. Randomness is unpredictability with respect to the intended theory and measurement.

What are the relations between indeterminism and randomness so construed? Probability theory doesn't answer this question. Algorithmic information theory (Calude 2002[2]; Downey and Hirschfeldt 2010) is a mathematical theory that deals with randomness in "its individuality", i.e. not only as a statistical phenomenon; we will use it in what follows.

Consider first the characteristics usually associated with randomness. Intuitively, randomness is identified with unpredictability (see Eagle 2005), lack of correlations (irregularity) and typicality. These characteristics of randomness can be tested by considering examples of "random" events, like coin-tossing. For example, a sequence of coin tosses looks very irregular, and no matter how many times we have tossed the coin, even thousands and thousands of times, predicting the outcome of the next toss seems impossible. We toned down the last sentence by saying that prediction of coin-tossing "seems" impossible. This is because, in principle, coin-tossing is as predictable as the motion of the planets, once the initial conditions are given. However, we "believe" that prediction is impossible – and this feeling is confirmed by experiment – because of the peculiar combination of circumstances of coin tossing, more precisely, the sensitive dependence on (some set of) initial conditions coupled with the inability to know these conditions with infinite precision.[9] Coin-tossing randomness is a simple example that shows that determinism is compatible with some forms of randomness.

Stronger forms of randomness exist from a mathematical point of view, but are they "available", can one produce them? The best bet is to consider quantum randomness. In (Abbott *et al.* 2015b) a non-probabilistic model for unpredictability is proposed and a quantum experiment E in which every individual outcome is maximally unpredictable is described. On the surface, having on hand a repeatable maximally unpredictable event is enough to claim that "quantum behaviour under measurement is truly random", in

[8] No less problematic than characterising a random event as the result of objective chance (taken as an intrinsic feature of the world) is identifying indeterminism with randomness (see Doyle 2011 for a historical review).

[9] We can assume that the coin-tossing is done by a mechanical device not subject to quantum mechanical vagaries (or the vagaries of an agent with free will).

the sense that no correlations exist between successive measurement results. Such a claim would be false, however. Indeed, the notion of "perfect/true/real/genuine randomness" is mathematically vacuous (Graham and Spencer 1990; Calude 2002[2]). In particular, in every sequence of zeros and ones there are correlations, so no such sequence can be truly random. There exist only degrees of randomness, with no upper limit. Repeating indefinitely the experiment E produces a highly incomputable sequence, so its degree of randomness is higher than coin-tossing randomness (Abbott *et al.* 2012 ; Diaconis *et al.* 2007).

Is indeterminism necessary for randomness? The answer is negative. Coin-tossing and software-generated randomness are (weak) forms of randomness which are algorithmically produced – no form of indeterminism is required. In fact both algorithmic information theory (Calude 2002[2]; Downey and Hirschfeldt 2010) and the practice of generating randomness show that randomness (of any form) is: a) defined by avoiding some algorithmically defined classes of patterns and b) produced algorithmically. Here are two examples, one mathematical and one physical.

The halting probability of a universal self-delimiting Turing machine U is called the Omega number of U and denoted by Ω_U (see Calude 2002[2]; Downey and Hirschfeldt 2010). The infinite sequence given by the binary expansion of Ω_U is uniquely determined by U. However, this sequence is "highly random" – technically, Martin-Löf random – because it passes all Martin-Löf tests of randomness, i.e. it avoids all patterns defined by Martin-Löf algorithmic model of randomness. Randomness appears when we "don't know" that the sequence is the binary expansion of the Omega number of U. This phenomenon is general because in every sequence there are patterns and correlations (Graham and Spencer 1990).

In the same way, in quantum mechanics the Schrödinger equation describes how the quantum state of some physical system changes with time: the wave function evolution is deterministic. Quantum randomness appears when we observe/measure certain individual quantum observables, for example, those that are *value indefinite* (Abbott *et al.* 2015a).

There is a loose analogy between the above two examples. The description of Ω_U by U corresponds to the Schrödinger equation. A first analogy appears when we compare the processes of computing the bits of Ω_U and measuring value indefinite observables in a quantum experiment E precisely described in Abbott *et al.* 2012. One can prove that the sequence of bits of Ω_U cannot be algorithmically computed (given U) and, similarly, the sequence of bits obtained by performing ad infinitum the quantum experiment E cannot be algorithmically computed (given full information about the experiment; Abbott *et al.* 2012; Id. 2015b). A second, more interesting analogy, appears when we compare the unpredictability of individual bits. An individual bit of Ω_U is maximally Martin-Löf unpredictable; a bit obtained in the quantum experiment E is maximally unpredictable in the sense of Abbott *et al.* 2015b.[10] These results are provable in both cases.[11]

Is randomness necessary for indeterminism? Again the answer is negative: there are cellular automata, as well as a plethora of computing machines, that work in non-deterministic ways without any use of randomness. The same is true if indeterminism is taken to be causal indetermininism. Given absence of causal determination in the production of a sequence of outcomes, nothing follows about what kind of regularity (or absence of regularity) will characterise the outcomes.

5. *Is randomness incompatible with freedom?*

Having seen that true randomness can be proved not to exist even mathematically, let alone in nature, we can now examine the compatibility between randomness and freedom more closely. The first point to make is that all arguments against the compatibility between free will and randomness based on "pure chance", "pure randomness", "true randomness" are simply unsound because they rest on a vacuous concept. An example is the argument based on Hume's and Schlick's ontological thesis according to which there is nothing intermediate between chance and determinism. In Eddington's words (1939: 182):

> There is no half-way house between random and correlated behavior. Either the behavior is wholly a matter of chance, in which case the precise behavior within the Heisenberg limits of uncertainty depends on chance and not volition. Or it is not wholly a matter of chance, in which case the Heisenberg limits […] are irrelevant.

Popper (1972: 227) disagreed:

> Hume's and Schlick's ontological thesis […] seems to me not only highly dogmatic (not to say doctrinaire) but clearly absurd.

Note that the reference to "correlated behavior" shows that Eddington was working with the product notion of randomness (or perhaps an amalgam of

[10] The Schrödinger equation cannot give the exact result of an individual measurement.

[11] The unpredictability/randomness of *every* individual quantum outcome was conjectured/postulated by Born (1926).

the process and product notions). And as we have seen, randomness in that sense only permits talk of degrees of randomness. Our choices and actions are never the product of "pure randomness".

A more interesting, but still unsound, argument for the incompatibility between free will and randomness is the following: randomness exists (in various degrees), so if an agent's actions are caused by randomness, the agent lacks control, so the assumption (C) is violated. In more detail the argument runs as follows.

1. Assume (P).
2. An agent *A* has free will with respect to *O*(*A*) in the context C if the assumption (C) is satisfied.
3. So *A* has ultimate control of which object in *O*(*A*) to choose.
4. If the object was chosen randomly (to some degree), then no one had full control of which object in *O*(*A*) was chosen.
5. Hence, *A* cannot not have full control on which object in *O*(*A*) to choose.
6. Therefore, *A* has no free will with respect to *O*(*A*) in the context C.

The argument may have some force if one could offer a clear explanation of how the object selected was chosen sufficiently randomly to prevent *A* having control over its decision. Unfortunately, this explanation is not given. Rather, it is often claimed that (P) is inherently able to provide this (cf. Almeida and Bernstein 2003; Strawson 1994). As we discussed in the previous section, however, randomness does not "float around", and is not something that is somehow "imposed on the agent". Randomness is just produced, and then used by the agent in coming to a decision. To make random decisions the agent needs to use a random generator, which is a device producing random bits of a certain quality (but never "truly random bits"); there is no alternative.

According to our proposed definition of free will, the detailed process used by the agent *A* to choose an object in some context C where more than one object is available for selection is irrelevant: with (P) in place, all that is needed for her to choose freely is satisfaction of condition (C). For the purposes of making a decision, using a random generator is no different than using the advice of a friend or getting more information from Wikipedia!

Still, for clarification we will look closely at the process of choosing at random. We henceforth assume that (P) is satisfied and consider first how a random generator may interact with an agent's decision making process, and second how the quality of the randomness generated may affect the agent's freedom. With the former in mind, we discuss four possible cases of interactions between the agent and the random generator. Suppose that in a context C, $O(A) = \{0, 1\}$, A chooses a random generator G with two possible outputs, 0 or 1, and then operates as follows:

(0) A uses G which outputs x, but ignores the output and picks an element of $O(A)$.
(1) A uses G which outputs x, and, after first deciding whether to use x to pick an element of $O(A)$, A then picks an element of $O(A)$.
(2) A uses G which outputs x and continues as follows:
- A uses x to determine whether to use G or not,
- depending on x, A makes no use of G or uses G to produce another (independent) output y which becomes its decision (in the last case G actually takes the decision on A's behalf).

(3) A uses G which outputs x and its decision is x (G is used to take the decision on A's behalf).

In case (0) it is clear that A's freedom is not hindered by randomness. Still it is worth pointing out that there was a random element in her decision process (though it did not impact A's decision). To show that case (1) does not undermine A's freedom, we restate that the information which A uses to make a decision is irrelevant to A's decision being free. A merely asks G for advice. In fact, cases (0) and (1) are identical if A picks something other than G's output. In either of these cases, however, A can consistently choose any element in $O(A)$ and, regardless of what G outputs, A has the final say on the decision. Hence, neither of these cases will disturb A's freedom. Cases (2) and (3) are more difficult to analyse. Case (2) is a hybrid between the earlier cases and the substantially more severe case (3). In (2) A operates G to generate a random bit. Depending on what this generated bit was, G either stops (leaving A to make the decision) or G is used to generate another random element of $O(A)$, which thereby becomes the "chosen" element. Thus (2) will either reduce to (0) or (3) depending on the result of the first computation. The point is that in cases (2) and (3), once A starts G, which object is chosen may potentially be decided by G rather than A. Is (C) fulfilled in these cases? Does the quality of random bits matter (e.g. if they form an incomputable sequence)?

The reason these cases seem to violate assumption (C) is because the agent gives up its final decision, not because the agent gives up its final decision to a random process. Asking another agent to make a decision on its behalf is no different than asking a random generator. Notice that whether A retains its freedom in asking another agent B to make its decision, is a delicate issue, one which is, in practice, judged on a case by case basis. This shows that from the point of view of free choice, the role of *B* is as detrimental to *A*'s freedom as the role of *G*. Giving up freedom to randomness is as harmful as giving up freedom to any other agent: *A* retains its freedom when it gives up its decision to *B* if and only if *A* retains its freedom when it gives up its decision to *G*.

Similarly, even if *A* does not voluntarily give up its decision, another agent *B* choosing an element of *O*(*A*) is no different than some random generator *G* doing the same. Thus, the fact that indeterminism allows for randomness should not lead us to conclude that free will is impossible any more than the fact that there are other free agents which are capable of choosing on behalf of others.

6. *Conclusion*

We have proposed a definition of free will that makes specific reference to agents, contexts and objects and is simple, two-stage and contextual (not absolute). We then pointed out that determinism is (almost certainly) incompatible with free will so conceived, and hence that a form of indeterminism is necessary – not sufficient – for free will. Quantum indeterminism illustrates such a form of indeterminism, although nothing in the paper hinged on seeing this as a way of accommodating free will. (The argument against understanding free will in quantum-mechanical terms is well known.) Crucially, however, we showed that one familiar worry about indeterminism of this or any other kind – its alleged ties to randomness, and the supposed incompatibility of free will and randomness in decision-making – does not apply. After showing that indeterminism and randomness do not imply each other, we provided a detailed analysis showing that randomness is not incompatible with free will. In short, we argued that indeterminism is a necessary condition for free will, and that, despite the threat from the notion of randomness, it is possible for agents to be free in some contexts under indeterminism. More precisely, we showed that randomness is compatible with free will so long as free will is itself metaphysically possible. We repeat that our arguments are relative and do not answer the main philosophical question about free will, namely does free will exist and, indeed, can it exist?[12] The exact meaning of this question is not obvious and,

[12] The guarded nature of this conclusion matches that found in a work like Thomas 2011, using a rather different definition of free will: "Although the Free Will Theorem can't prove if we have free will, it does have a fundamental consequence: if the Universe is deterministic, and a particle's behaviour is always described by a function of the past, then we can't have free will". The free will theorem roughly says that under three specific assumptions "If experimenters have free will, then so

even within the framework of the present paper, the question may be answered differently depending on how it is interpreted. An examination of the precise requirements for the existence of free will and whether those requirements are met is beyond the scope of this paper.

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