Analogy in the natural sciences: meeting Hesse's challenge

Paul Bartha

Abstract: Hesse's challenge, over a period of many years, was to provide a theory of scientific concepts and a logic of science, both of which were based on analogies and metaphors. We can equally well understand her challenge as the development of a theory of metaphor and analogy, with the natural sciences serving as an important *special case*. This paper explores two distinct ways in which we might see analogies in science as a special case in relation to a general theory of analogy.

On the *leading special case* view, understanding how analogies work in science is the key to developing a general theory. On the *limiting case* view, providing a general theory, especially of how analogies work in everyday contexts, is a precursor to understanding their specialized role in science. While both approaches are present in Hesse's work, I suggest that the former is associated with her early (1966) work on analogical arguments and the latter with her later (1974; 1988) theories of metaphor and meaning. Her shift towards the limiting case view is associated with growing pessimism about the prospects for inductive logic. Yet the distinction remains important in current work on analogy: Hesse's challenge is to reconcile normative theories of analogical reasoning with computational models of analogical cognition.

Keywords: analogy; analogical reasoning; metaphor; Mary Hesse.

We rely upon analogies and analogical reasoning all the time. We use them to solve everyday problems. We use them in sophisticated legal reasoning, in mathematics, in the sciences and in many other settings. Some of this reasoning is presented in explicit form, as an *analogical argument* in support of a particular conclusion. The logic of analogical arguments has been studied since the time of Aristotle, albeit with limited success.[1]

Analogies have particular importance in the natural sciences, where they play many roles. They are a powerful *heuristic* tool, as we know from examples

[1] See *Topics* 156b10, *Rhetoric* 1402b15 and *Prior Analytics* 69a1. For Aristotle, there are two distinct argument forms that might be called analogical: the *argument from example* and the *argument from likeness*. The first belongs both to rhetoric and inductive logic, the second only to rhetoric. For an excellent study of analogical reasoning in antiquity, see Lloyd 1966.

that range from Maxwell's discovery of the displacement current[2] to the discovery of aspirin (Collier 1984). Analogies between disparate phenomena can lead to systematization or *conceptual unification*, as in the case of electricity, where scientists came to acknowledge a "common nature" found in lightning, the Leyden jar and "animal spirits" (Pera 1992). Analogical arguments play an important *logical* role by providing justification for scientific hypotheses in at least two circumstances where conventional evidence is scanty or unavailable: establishing the plausibility of a novel hypothesis, and providing justification for hypotheses in fields such as archaeology where we lack direct means of testing.[3]

In thinking of these many functions, it is important to distinguish between the broad role that analogies can play in guiding a sustained research program and the narrow role of providing the basis for an individual argument. In the former role, analogies behave like metaphors.[4] They provide striking models and images that shape our perception of phenomena. These models are dynamic: they evolve as our concepts change, as illustrated by the example of electricity. By contrast, individual analogical arguments, aimed at the specific extension of some known result, pre-suppose stable meanings.[5] The tension between these two roles, *semantic* and *logical*, is a major theme in this paper.

In the 19th century, works on philosophy of science gave considerable prominence to the place of analogies and analogical arguments in the logic of science.[6] In 20th century logical empiricist models of theoretical confirmation, analogies were much less central. In recent years, the fortunes of analogy have risen once again. Apart from studies within the history and philosophy of science, research on analogy has been transformed by work in psychology and computer science.[7] Most of this research, however, is not oriented towards the concerns of inductive logic.

[2] Maxwell's discovery, outlined in a series of papers contained in Maxwell 1890, is the subject of a great deal of scholarly work within which the role of analogy is disputed. See Hesse 1973, Siegel 1986 and 1991, Morrison 2000, Nersessian 2002.

[3] See Bartha 2010 for discussion of these and other examples.

[4] It is difficult to distinguish between analogy and metaphor. Broadly, an analogy is any comparison between two systems of objects that highlights respects in which they are thought to be similar. Typically, we think of analogy more narrowly as an explicit and precise enumeration of similarities, while a metaphor ("Heat is a fluid") is a high-level comparison that conveys similarities in an evocative and open-ended fashion. In terms of Hesse's two-fold project of providing a theory of scientific concepts and a logic of scientific inference (see below), analogy is linked most closely to logic and metaphor to meanings, though metaphors clearly shape our inferences.

[5] For further discussion of this contrast, see Stepan 1996 and Bartha 2010: 11-12.

[6] See Snyder 2006 for a discussion of the role of analogy in the inductive logic of Whewell, Herschel and Mill.

[7] For anthologies of multi-disciplinary work on analogy, see Helman 1988, Gentner *et al.* 2001 and Kokinov *et al.* 2009.

At the risk of oversimplification, the objective of most recent research is to capture the fundamental cognitive processes involved in analogical reasoning: *retrieval* of a source analogue, *mapping* between elements of the source and target, *transfer* of information from source to target, and *learning* of new concepts.[8] Computational theories implement analogical reasoning in computer programs inspired by connectionist or case-based reasoning models of human thinking.[9] Such models apply equally well to scientific and non-scientific analogies; indeed, they are often motivated by problem-solving tasks outside science. These computational theories are geared towards *generating analogical inferences* rather than *evaluating analogical arguments* (let alone constructing an inductive logic).[10]

The shift in research orientation prompts a question that is the reverse of the traditional one. Rather than seeking to understand the role that analogies play in scientific theories, we can ask: *what is the role of the sciences in a theory of analogy*? More sharply put: how should a special theory of analogies in the natural sciences relate to a general theory of analogies?

In order to refine this question, let us turn to the work of Mary Hesse, who has been thinking about analogies for over fifty years. One thread that runs through Hesse's work is the idea that the *natural sciences* constitute a particularly important special case for understanding analogies. Assuming that Hesse is right, our question becomes:

> *How* are the natural sciences an important special case for understanding analogies?

I shall argue that Hesse offers two distinct answers to this question. In her 1966 book, *Models and Analogies in Science*, her view is that the natural sciences are a *leading special case*. A quarter century later, in "The Cognitive Claims of Metaphor" (Hesse 1988a), her view has shifted: the natural sciences are a *limiting special case* for understanding analogical (and metaphorical) reasoning.[11] In this paper, I explain these two perspectives in Hesse's work. I then

8 See the introduction to Gentner *et al.* 2001. A similar decomposition is offered in Holyoak and Thagard 1996 and elsewhere.

9 Again, some of this work is discussed in Bartha 2010. For further examples, see the anthologies listed in footnote 5.

10 Computer models of legal reasoning are a notable exception; see Ashley 1990. To be sure, computational theories offer criteria for assessing analogical arguments indirectly, in terms of constraints such as *systematicity* of the analogical mapping (Gentner 1983). See also Nersessian's paper in this volume. My point here is only that the representation of analogical reasoning as a cognitive process has displaced its representation and assessment as an argument form.

11 As I clarify below, the shift is a matter of emphasis. Elements of the later view are apparent even in (1966); some elements of the earlier view persist in Hesse's later work, especially Hesse 1988b.

explore their implications for the relation between a special theory of analogies in the natural sciences and a general theory of analogy.

It is important to remember that Hesse, in (1966) and even more clearly in (1974) and (1975), was not just interested in a theory of analogy. Her objective was twofold: a theory of meaning for scientific concepts and a logic of science, with model-based reasoning as the engine driving both enterprises. Initially, "Hesse's challenge" (referring to the title of this article) was to develop these two programs more or less independently. As I shall argue, she came to believe (by 1974) that there was significant tension between the two projects, and that it did not bode well for inductive logic. Hesse's challenge, in (1974) and persisting in (1988a) and (1988b), morphed into something different: to save inductive logic from her own semantic theory. The shift that I am alleging in Hesse's position on analogy, from treating the natural sciences as a leading special case to viewing them as a limiting special case, occurs in tandem with these changes in Hesse's general stance on inductive logic.

I believe that the tension that Hesse noted persists in current debates about analogy. My main conclusion is that both perspectives on the role of the natural sciences are legitimate, but they lead to very different types of theory. Taking the natural sciences as a *leading special case* leads to a *normative theory* of *analogical arguments*, while treating them as a *limiting special case* leads to a *psychological or computational model* of the *cognitive processes* that comprise analogical thinking. The tension persists because we need both approaches, yet it is not easy to see how they can be reconciled.

I begin (§1) by explaining the distinction between leading and limiting special cases. In (§2), I trace the evolution of Hesse's theory of analogy, concentrating on what she identifies as the main philosophical challenges and on what she means when she describes the natural sciences as a "special case". As we shall see, Hesse moves from a normative theory of analogical arguments to a framework that makes it difficult to accommodate any normative account. The final part of the paper (§3) briefly reviews some current approaches to analogy and offers suggestions for reconciling the normative and psychological perspectives.

1. *Two types of special case*

The distinction between *leading special cases* and *limiting special cases* derives from the field of inductive reasoning in mathematics. I first explain how the distinction works in that setting, and then apply it to Hesse's work on analogy.

In a lucid discussion of inductive reasoning in mathematics, Polya notes that "generalization, specialization, and analogy often concur in solving mathematical problems" (1954: 15). This happens when a special case of some problem is analogous to the general case. Suppose we can solve the special case. We might then use the analogy in two different ways: in a modest plausibility argument that provides inductive support for the generalization, or (more ambitiously) in a rigorous proof of the generalization that employs or extends the very techniques used to prove the special case.

In Polya's terminology, a "leading special case" for a mathematical problem is a special case that is both "particularly accessible" and the key to proving the general case because "the other cases follow" (1954: 24). In other words, it is a special case which bears the best possible sort of analogy to the general case because both uses of analogy just mentioned – providing support for a general conjecture and furnishing the tools for its proof – are effective.

Polya illustrates the idea with an example:

> The area of a polygon is A, its plane includes with a second plane the angle α. The polygon is projected orthogonally onto the second plane. Find the area of the projection (1954: 24).

He notes that it is "especially easy to handle" one particular shape: a rectangle with sides a and b, when side a is parallel to the line formed by the intersection of the two planes. The area of the rectangle is (ab); the area of the projection (also a rectangle) is ($ab \cos\alpha$). For this special case, the area of the projection is $A \cos \alpha$. By analogy, it is plausible to conjecture that this result holds for any polygon. But the analogy supplies more than just a plausible conjecture: it also gives us the tools to prove it. First, we extend the result to right-angled triangles (by bisecting the rectangle into two such triangles), then to arbitrary triangles (by combining two right triangles), and finally to polygons (which can be decomposed into triangles). As Polya notes, "*the solution of the problem in the leading special case involves the solution in the general case*".

In other examples, however, a special case might have only the first of the two features noted above. Consider the following theorem of one-variable calculus:

> Let f be a continuous one-one function defined on an interval, and suppose that f is differentiable at $f^{-1}(b)$, with derivative $f'(f^{-1}(b)) \neq 0$. Then f^{-1} is differentiable at b, and $(f^{-1})'(b) = 1 / [f'(f^{-1}(b))]$ (Spivak 1980: 222).

The generalization of this result to functions of n variables is known as the *Inverse Function Theorem*, which states (roughly) that if the derivative of f is locally invertible, then f itself is locally invertible. The special case where n = 1 provides analogical support for the general conjecture, but the proof of the general case uses techniques and ideas that go well beyond the easy proof of the special case. I refer to an example of this sort as a "limiting special case". Although the special case may give us confidence that the generalization is true, the usual methods for solving the special case do not generalize. We might even say that we acquire a deeper grasp of the special case *after* proving the general case.

A slightly more general distinction between leading and limiting special cases, I suggest, is helpful in other settings. My concern in this paper is with analogical reasoning in the natural sciences as a special case of analogical reasoning in general. Any problem or challenge that we raise about analogical reasoning in general can be restricted to the natural sciences. With respect to any such challenge, the natural sciences are a *leading special case* if we can solve (or hope to solve) the general problem by extending the techniques and ideas developed for the natural sciences. They constitute a *limiting special case* if the solution to the general problem requires ideas drawn from external sources; these ideas can then be applied to the natural sciences.

How is all this relevant to Hesse's work on analogy? In keeping with her approach to philosophy of science in general, Hesse divides the problem of analogy into two broad categories: the *logical* problem of justifying analogical arguments and the *semantic* problem of accounting for analogical (or metaphorical) meaning and reference. In 1966, she conceives of the natural sciences as a leading special case with respect to the logical problem, and a limiting special case with respect to the semantic problem. Over the years, the semantic problem looms ever larger in her thought. The leading-case perspective appears less and less tenable and the solution to the logical problem becomes correspondingly remote. The next section outlines this trajectory in more detail.

2. *Hesse on analogy: an evolving challenge.*

Throughout her philosophical career, Hesse has identified, and attempted to answer, fundamental philosophical questions about analogy. Although the questions have remained broadly similar, the specific concerns that she emphasizes have changed. This section reviews some of the main changes, concentrating on (1966), (1974) and (1988a). The objective is to flesh out the claim made at the end of the previous section: as Hesse devoted increasing attention to semantic issues, her early project of developing a normative theory of analogical reasoning based on the natural sciences became less and less feasible. At the end of this section, however, I suggest that there is still room for such a project in Hesse's later philosophy of science.

1966: Models and Analogies in Science

In (1966), Hesse focuses almost exclusively on analogies in the natural sciences. She raises two major philosophical challenges. The first, which she calls the traditional 'problem of analogy', has two parts: to identify criteria for good analogical arguments and to provide a philosophical justification for analogical reasoning. In short, the problem is to develop and defend a substantive normative account of analogical arguments. The second challenge is to understand the semantic role of analogies and metaphors. In (1966), this amounts to explaining the meaning of theoretical concepts (and the structure of scientific explanation) in terms of models, with the relation between model and theory construed as metaphoric.

Hesse meets these challenges with varying degrees of success. Chapter 2 of the book proposes an admirably clear framework for evaluating analogical arguments. Hesse offers a tabular representation for analogical arguments in which the known similarities and differences of the two objects (the *source* and *target domains*, to anticipate later terminology) are drawn up side by side. Hesse's simplest example is a comparison between Earth and the moon (1966: 59) that could be drawn up by somebody wondering whether there might be humans on the moon:

EARTH	MOON
spherical	spherical
atmosphere	no atmosphere
humans	?

Inter-domain relations of similarity and difference are *horizontal relations*; intra-domain relations, typically causal in nature, are termed *vertical relations*. The *positive analogy* consists of known similarities (spherical), the *negative analogy* consists of known differences (atmosphere), and the *neutral analogy* consists of the features of the source not yet known to hold or fail in the target (humans). Equipped with these definitions, Hesse proposes a compact set of requirements for an acceptable analogical argument:

1. *Requirement of material analogy.* The horizontal relations must include observable similarities.
2. *Causal condition.* The vertical relations must be causal relations "in some acceptable scientific sense" (1966: 87).
3. *No-essential-difference condition.* Essential features of the source domain must not be known to belong to the negative analogy.

These requirements are a fair approximation of ordinary norms of analogical reasoning, but I will not discuss or evaluate them here.[12] It is enough to notice that the three requirements are primarily based on and supported by discussion of analogical arguments in science. Hesse even anticipates difficulties in extending the theory to non-scientific analogies (e.g., analogies in religion).

Hesse tackles the justification of analogical reasoning in chapter 3 of her book (1966) and in a companion paper (Hesse 1964), with limited success. As she conceives it, the problem is to extend the inductive logic of the day, specifically Carnap's account of confirmation, to analogical arguments.[13] She is openly skeptical about the prospects of this approach in her book (1966: 55 and 117), but more sanguine in (Hesse 1964). There, she argues that while Carnap's λ-system accounts for next-case induction but fails to account for analogical arguments, the η-system (Carnap and Stegmuller 1959) might offer "an adequate explication". Once again, without evaluating Hesse's position, let us note that this aspect of her theory is explicitly linked to scientific analogies.

In the remainder of the book, Hesse offers a preliminary, but very fruitful, analysis of the semantic role of models and analogies in science. She begins with a familiar objection to Hempel's deductive-nomological model of scientific explanation: the valid derivation of an *explanandum* stated in observational terms from an *explanans* that includes theoretical laws (stated in theoretical terms) requires the introduction of problematic correspondence rules (1966: 174). Hesse's solution is to deny the distinction between theoretical and observational concepts: "there is *one* language, the observation language, which like all natural languages is continually being extended by metaphoric uses and hence yields the terminology of the explanans" (1966: 175). The "theoretical" concepts of the explanans acquire their meaning metaphorically. Hesse admits that "we are far from understanding this process," but she takes her cue from Black's "interaction view" of metaphor (Black 1962). Black rejects the idea that metaphorical language can be reduced to literal language and substitutes the holistic view that two systems linked by a metaphorical comparison "seem to interact and adapt to one another" (Hesse 1966: 163). Hesse argues that the same type of interaction occurs in analogical modelling. She writes: "to see the problem of 'meaning of theoretical concepts' as a *special case* of [understanding how metaphors are introduced and applied and exploited] is one step in the solution of this problem" (1966: 175; italics added).

12 See Bartha 2010 for discussion.

13 Hesse also considers other approaches, such as Hempel's theory of confirmation and Popper's account of falsifiability.

Once again, I shall not evaluate Hesse's substantive position. Note, however, the sharp contrast between Hesse's approach to *analogical meaning* in science, which is based on the application of ideas drawn from an external field (the philosophy of language), and her approach to *justification* and *inductive logic*, which proceeds entirely from scientific examples and more or less orthodox approaches within the philosophy of science. The natural sciences are thus a leading special case for Hesse's normative theory of analogy, and a limiting special case for her emerging theory of meaning. To a large extent, she handles the two problems independently.

1974: The Structure of Scientific Inference

The Structure of Scientific Inference differs from the earlier work in a number of ways. In the first place, the scope of the book is broader: Hesse's aim is not just to develop a theory of analogy but also "attempting, against all current odds and fashions, to develop a *logic* of science" (1974: 6). Second, the book takes into account recent "radical criticism" of positivist philosophy of science, the work of Feyerabend, Kuhn and others who appear to reject inductive logic and even the "analytic style" of logical empiricism. From these thinkers and from the work of Quine, Hesse takes the problem of "meaning variance": scientific concepts, *both observational and theoretical*, have historically unstable meanings. Hesse takes meaning variance very seriously; it motivates much of her later work.

The main challenges for a theory of analogy (and more generally for the "structure of scientific inference") remain broadly the same as in (1966): the semantic problem of accounting for the meaning of scientific concepts, and the logical problem of finding a justification for analogical (and other types of) inference. Despite Hesse's stated aim of developing an inductive logic, *The Structure of Scientific Inference* is largely devoted to developing her semantics of concepts and exploring its implications. She vastly expands her earlier ideas on metaphor into the "network model" of meaning in which the language of science is "a dynamic system which constantly grows by metaphorical extension of natural language" (1974: 4). She sees this as the best response to the problem of meaning variance.

The implications of this model for logic, however, are severe. Hesse advocates a broadly Bayesian epistemology that uses personal probabilities to represent degrees of belief. But it is unclear how this epistemology meshes with her "network model" in which no concepts have stable meanings. In an essay from the same time period, "Bayesian Methods and the Initial Probabilities of Theories," Hesse concludes that there is no "global inductive theory", but only "elementary inductive inference applicable when the language base remains sufficiently stable" (Hesse 1975: 104). In fact, she accepts only two legitimate forms of "elementary inductive inference": next-case induction and analogical inference.

If we restrict our attention to analogical inference (more specifically, analogical arguments), we find that Hesse makes little headway between (1966) and (1974). In one respect, the later book is actually *less* ambitious than the earlier one: Hesse does not refine, or even mention, any requirements for good analogical arguments. She does explore the justification of "elementary inductive inferences", in both (1974) and (1975), via her Clustering Postulate:[14]

> Of *r* instances of *P*'s, it is more probable that none or all will be positive instances of 'All *P*'s are *Q*'s' than that there will be any other proportion (1975, 94; see also 1974: 154).

Echoing her analysis in (1966), Hesse shows that this assumption provides a basis for next-case induction but not for analogical inference. Just as in her (1964), she mentions that Carnap's η-system accommodates some types of analogical inference. But far from endorsing it, she writes: "practically nothing is known about the mathematical properties of the combinatorial algebra", and hence it is "not profitable, at least at present, to pursue such an approach to analogical argument" (1975: 170). The discussion is inconclusive.

In (1966), Hesse saw the logical and semantic problems of analogy as separable. In (1974) and (1975), this is still true, although as we saw, Hesse's concerns about meaning variance lead to a stripped-down inductive logic with just two elementary inference patterns. Only these two 'finitistic' argument forms make sufficiently weak demands on the stability of linguistic meaning. She writes:

> In spite of much recent discussion of 'meaning variance', I presuppose that language, as well as its meanings, remains locally invariant throughout such pieces of inference, for if it does not it is impossible to understand how communication, let alone rational argument, is possible either in science or everyday life (1975: 104).

The idea that local meaning invariance is crucial to inductive logic raises an interesting question: why did Hesse take from Kuhn the problem of meaning variance (across paradigms) but ignore his conception of normal science? Normal science, with its extended periods of stability, appears to offer a haven for inductive logic and Bayesian epistemology.[15] Hesse published a favourable review (Hesse 1963) of *The Structure of Scientific Revolutions* which barely mentions normal science. In (Hesse 1974), she cites Kuhn just four times. Three of

[14] Hesse explicitly acknowledges Keynesian and Carnapian influence.

[15] In addition to Kuhn's own work, Salmon (1990) proposes a reconciliation of Bayesian and Kuhnian ideas, though arguably one that applies only to normal, rather than revolutionary, science.

these citations simply link Kuhn to the problem of meaning variance, while the fourth is a reference to his book on the Copernican revolution. As far as I can tell, not once does Hesse discuss normal science. I shall return to this point following discussion of her 1988 papers.

1988: The Cognitive Claims of Metaphor

At the conclusion of her essay, "The Cognitive Claims of Metaphor," Hesse writes:

> The understanding of metaphor therefore poses a radical challenge to contemporary philosophy (Hesse 1988a: 14).

"Hesse's challenge" in 1988 is to explain how metaphoric concepts refer and how metaphoric statements have truth-values. The challenge is radical because, in Hesse's view, "all language is metaphorical" (1988a: 1). As in her earlier work, Hesse rejects any reduction of metaphorical to literal language. Instead, she argues that metaphor is the fundamental notion:

> Literal sense and 'ordinary descriptive reference' then become the limiting cases of language-use appropriate for everyday and scientific commerce with the natural environment. [...] There is therefore no ideal literal sense by means of which all-pervasive metaphor is to be constrained (1988a: 13).

The challenge that Hesse has identified as fundamental falls entirely within the scope of her semantic project.

What about inductive logic? Even more clearly than in (1974), logic is subordinate to the theory of meaning:

> Now while neglect of meaning-change may be acceptable in purely formal contexts where no relation to the empirical world is in question, it must be regarded as seriously distorting where the languages of everyday description are concerned (1988a: 1).

Hesse has abandoned the idea of treating the natural sciences as a leading special case, a source of special insight into analogical reasoning and inductive logic. Inductive logic must now be understood through the lens of her "network theory" of meaning, as a limiting case:

> I say literal sense is a 'limiting case' here, in conformity with the general thesis that all language in use is necessarily metaphorical, although the scientific truth-criteria of prediction, test, and self-correction permit the definition of ideal limiting notions of the 'literal' and even of 'correspondence' truth (1988a: 13).

In another paper published in the same year, "Theories, Family Resemblances and Analogy" (Hesse 1988b), Hesse spells out very clearly the problem that her network or *family-resemblance* (f.r.) theory of meaning generates for logic, whether deductive or inductive:

> In terms of an f.r. theory we have to regard a working language as a sequence of sets of assignments of objects to predicate classes, with transformations between one assignment and the next which are functions of changes of evidence, perception and interest... Within such a confusion of changing meaning, how is the possibility of logical inference to be restored? (1988b: 327)

She takes the problem seriously enough to reject both deductive logic and, as in (1974), all forms of inductive argument apart from next-case induction and analogical inference. Just as before, she argues that these two finitistic inference forms presuppose just enough meaning stability for communication and no more ("a level of stable commonsense objects and their properties has to be presupposed… and is justified by success in prediction and communication").

The normative dimension of Hesse's theory of analogy has all but disappeared. There is no discussion of criteria that distinguish between good and bad analogical arguments. There are no examples of scientific analogies. Furthermore, Hesse is more pessimistic than ever about the prospect of a philosophical justification for analogical inference:

> And just as the old problem of inductive "justification" cannot be solved except by stating the conditions in which we assume it valid, so analogical inference cannot be justified except by stating the conditions that have to be presupposed if the inferences we in fact make are to be valid. These conditions constitute what I have called the clustering hypothesis… (1988b: 332).

Although Hesse ends her paper with a call for "development of a confirmation theory for scientific inference to take account of the f.r. characterization of concepts" (1988b: 337), it is difficult to see how such a project is feasible.

I have linked Hesse's shift in attitude towards the natural sciences with a retreat from the project of providing a normative theory of analogy. To close this section, I mention two responses that one might offer to Hesse, amounting to distinct avenues for research on analogy.

The first response is to embrace the naturalism of Hesse's later work. As Hesse puts the point:

> Metaphoric meaning and analogical reasoning have now become issues within AI in terms of programs for problem solving. Part of the reason for this has been the sterility of attempts within logic and linguistics to account for the cognitive processes that are fundamentally analogical rather than deductive. It has long been obvious that the human problem solver does not generally think deductively or by exhaustive search of logical space (1988b: 317-18).

Many people doing research on analogy would see nothing wrong with abandoning the quest for a normative theory and turning to the study of the cognitive processes that comprise analogical reasoning. Many would applaud this direction in Hesse's thinking and perhaps urge her to drop the lingering attachment to confirmation theory or inductive logic. Such researchers tend to share the later Hesse's view that the natural sciences are a *limiting case* rather than a *leading case* for understanding analogical inference. To understand analogical reasoning is to model its component cognitive processes, which are not peculiar to science. These cognitive processes involve fluid mappings and evolving categories, characteristics of creative thinking, just as in Hesse's network theory. Such theories are not meant to be normative and cannot be normative, since the very same cognitive processes can produce both good and bad analogical arguments.

But there is an alternative response. We can part company with Hesse at a point where it is still possible to develop a normative theory of analogy. The most obvious strategy is to point out that Hesse has overstated the problem of meaning variance. Her requirement that meanings remain stable over the course of a single (analogical or inductive) inference is too thin to provide a platform for any scientific community. An appealing alternative here, especially given the period during which Hesse was developing her ideas, is Kuhnian "normal science". In the 1969 Postscript to *Structure of Scientific Revolutions*, Kuhn identifies analogical reasoning as the engine of expansion for normal science. He writes:

> Scientists solve puzzles by modelling them on previous puzzle-solutions, often with only minimal recourse to symbolic generalizations (Kuhn 1970: 189-90).

A commitment to a Kuhnian paradigm, here construed as a loosely defined set of core principles, methods and relatively stable concepts, is indispensable to the practice of normal science. A normative account of individual analogical arguments might be feasible within the context of normal science.[16]

If we take this route, many questions still remain. I single out three. What are the most hopeful options for a normative theory of analogy? What is the connection between such normative accounts and the view that the natural sciences constitute a *leading case* for understanding analogies? Finally, how can any normative analysis be reconciled with the "cognitive processes" approach that takes shifts in meaning as an essential feature of analogical reasoning?[17]

[16] I single out Kuhn for his salience during the period of Hesse's work on analogy, but Lakatosian research programs, with their stable cores, offer a different avenue for developing this type of response (Lakatos 1970).

[17] There is another excellent question: how can there be a normative account of "revolutionary" analogies, since there are no stable meanings in a scientific revolution? I set this question aside.

3. *The challenge since 1988*

An impressive array of computational theories of analogy has emerged since the 1980's. The introduction to *The Analogical Mind*, an anthology of essays on analogy (Gentner, Holyoak and Kokinov 2001), offers a picture of how many cognitive scientists now understand the central issues about analogical reasoning. These issues are structured around a widely accepted decomposition of the process of analogical reasoning into four stages:

1) *Retrieval*: finding a relevant "source" domain.

2) *Mapping*: establishing a set of systematic correspondences between the elements of the source and target domains.

3) *Analogical inference*: transfer of information from source domain to target domain.

4) *Meaning change*: adopting new schemas or concepts.

There is wide variation among these computational theories. There are connectionist programs, beginning with the *structure-mapping theory* due to Gentner and her colleagues (Gentner 1983; Forbus, Ferguson and Gentner 1994; Forbus 2001), the multiconstraint theory due to Holyoak and Thagard (1989; 1995), and more sophisticated successors (Eliasmith and Thagard 2001; Hummel and Holyoak 2003). There is Hofstadter and Mitchell's COPYCAT program (Hofstadter 1995), which offers a distinctive connectionist model of analogical reasoning. There are case-based reasoning programs, such as Ashley's HYPO (Ashley 1990), and Carbonell and Veloso's PRODIGY (Carbonell 1986; Carbonell and Veloso 1993).

I believe that these computational theories re-define Hesse's challenge in an interesting way. The four-part decomposition of the cognitive process yields a set of challenges that can be considered successors to what Hesse called the semantic problem of analogy. They give that problem a computational interpretation, focused on concrete issues of representation, mapping and meaning change. For example, Hofstadter writes, "no true-life situation comes with hard and fast boundaries" (1995: 69), and his COPYCAT program deliberately implements concepts with "fluid boundaries".

What about the other half of Hesse's challenge, the logical problem of analogy? There appears to be no place for explicit analogical arguments in the decomposition. But the logical problem nevertheless remains relevant. This is clear because explicit analogical arguments are regularly developed and criticized in many contexts, most notably in the law, mathematics and the natural sciences. In the remainder of this paper, I sketch what I take to be three broad options for developing a normative account of analogy. I assess the connection between one of these options and the claim that the natural sciences constitute a suitable leading special case for understanding analogical arguments. Finally, I offer a suggestion for relating the logical problem of analogy to the computational theories.

There are three levels of generality at which norms of analogical reasoning might be proposed.

1. We might seek *universal* norms of analogical reasoning and a correspondingly universal 'logic of analogy'. Carnap and Stegmuller (1959), discussed earlier, represents an approach of this sort. The Carnapian strategy has been developed further by Niiniluoto (1981; 1988), Kuipers (1984; 1988) and Festa (1997), and more recently by Maher (2001) and Romeijn (2006). These frameworks incorporate explicit parameters to represent judgments of similarity.

It is important to distinguish between approaches in the axiomatic tradition of logical probability and Bayesian modelling with subjective probabilities. In the second case, the objective is to develop a framework for representing analogical arguments in Bayesian terms. De Finetti's work on partial exchangeability (de Finetti 1980) represents such an approach, as do some of the papers just cited. Romeijn (2006) distances himself from the axiomatic tradition, describing his objective as "providing statistical models that underlie the analogical predictions":

> It may be too ambitious to aim for the definitive class of all rational probability assignments that capture analogical considerations. It is more in line with an emphasis on local inductive practice, as recently discussed in Norton (2003), to propose a collection of models only, and to decide about the exact nature of analogical predictions on a case by case basis… (Romeijn 2006: 265-266).

The framework is universal, but the only universal norms are norms of Bayesian reasoning. Romeijn shares Hesse's pessimism about a substantive global inductive logic.

2. Following Norton (2003; 2012), we might hold that there are only *local* norms for evaluating analogical arguments. Norton believes that the project of analyzing analogical reasoning in terms of formal schemata is doomed:

> If analogical reasoning is required to conform only to a simple formal schema, the restriction is too permissive. Inferences are authorized that clearly should not pass muster… The natural response has been to develop more elaborate formal templates… The familiar difficulty is that these embellished schema never seem to be quite embellished enough; there always seems to be some part of the analysis that must be handled intuitively without guidance from strict formal rules (2012: 1).

In keeping with his "material approach" to inductive inference, Norton (2012) argues that there is no universal logical principle that "powers" analogical inference "by asserting that things that share some properties must share others". Instead, each analogical inference is warranted by some local constellation of facts about the target system that he terms "the fact of analogy". These local facts are to be determined and investigated on a case by case basis.

3. Finally, we might think that there are 'regional' or *intermediate* norms: guiding principles for types of analogical arguments. A normative theory of analogical arguments consists of a classification and specific criteria for evaluating arguments within each class.

Hesse (1966) can be viewed as an attempt to develop a theory of this type, focused on explanatory analogical arguments within the natural sciences. Some case-based reasoning programs, such as Ashley's HYPO program (Ashley 1990), fall into this category. Ashley restricts his attention to trade secrets law. He identifies a set of relevant *dimensions*, features such as the existence of a nondisclosure agreement whose presence or absence is relevant in determining whether or not a trade secret has been violated. In (Bartha 2010), I distinguish several types of analogical argument in the sciences and develop norms for each category.

To motivate the idea that there are different kinds of analogical argument with distinct evaluation criteria, consider the difference between the following two analogical arguments.

Example 1 (Rectangles and boxes). Suppose that you have established that of all rectangles with a fixed perimeter, the square has maximum area. By analogy, you conjecture that of all boxes with a fixed surface area, the cube has maximum volume.

Example 2 (Priestley on electrostatic force). In 1769, Priestley suggested that the absence of electrical influence inside a hollow charged spherical shell was evidence that charges attract and repel with an inverse square force. He supported his hypothesis by appealing to the analogous situation of zero gravitational force inside a hollow shell of uniform density (Priestley 1769).

The first argument is based upon a mathematical analogy. The idea is that an entailment in one domain can be generalized so as to apply to the other domain. The second argument involves a type of explanatory analogy. Here the focus is on analogous *effects* in two domains, and the central idea is that they can be explained by analogous hypotheses. We might expect that different guidelines should be used to assess these very different arguments, and we might hope to find guidelines at a level of generality somewhere between Norton's "local facts" and Hesse's Clustering Postulate.

Each of the above three approaches to a normative theory of analogical arguments has merit. I suggest, however, that of the three, the "intermediate norms" approach is most closely linked to the strategy of formulating a normative theory on the basis of some leading special case or cases. We can accept Norton's basic point that there is no schema that decides the validity of each individual analogical argument. This observation is compatible with the existence of distinct classes of analogical argument, each amenable to guidelines that need not be fully precise. On the "intermediate norms" approach, a leading case provides the key not to finding some universal schema for analogical reasoning, but rather to identifying and evaluating a class of analogical arguments.

There can be many leading cases, corresponding to distinct types of analogical reasoning. In particular, analogical arguments in the law (or areas of the law) may be a valuable source of insight. I suggest, however, that there are good reasons to revive Hesse's (1966) insight that the natural sciences are an especially fruitful choice for developing a normative account of analogical arguments. These reasons are connected to characteristics of Kuhnian normal science, and I shall take for granted the suggestion (offered at the end of section 2) that we can avoid radical meaning variance by framing a normative theory of analogical arguments within the context of Kuhnian normal science.

The first point is that normal science and mathematics are puzzle-solving activities, and analogical reasoning is above all a tool for solving puzzles. Well-formulated puzzles have clear structure. The best solutions involve techniques that generalize and allow us to solve similar puzzles. It is no surprise that normal science, a premier puzzle-solving enterprise, is an excellent choice of leading special case for a normative theory of analogy.

The second point is that within natural science and mathematics, we have a rich and varied set of plausible analogical arguments. There are mathematical and explanatory analogies (as illustrated by Examples 1 and 2), analogical arguments used to make predictions, and analogical arguments used to change or extend scientific categories. Starting from such examples, we can obtain a comprehensive family of types of analogical arguments, each with distinctive norms.

4. *Conclusion*

Hesse was one of the first people to recognize the tension between two fundamental aspects of analogy. On the one hand, analogical reasoning is a highly creative form of reasoning that depends upon fluid concepts and the ability to recognize novel types of similarity. In this respect, it relies upon cognitive processes that are not specific to scientific reasoning; their application to scientific contexts is a limiting case. On the other hand, analogical arguments in science play an important role in theoretical confirmation. Such arguments are regularly proposed, criticized, and then developed further or discarded. So a theory of analogy requires norms such as those proposed by Hesse in (1966). Furthermore, the Bayesian model of confirmation, the framework within which Hesse hoped to locate analogical arguments, requires stable meanings. Both of these considerations suggest that if our objective is in part a normative theory of analogical arguments, the natural sciences hold promise as a leading case.

Hesse never lost sight of the importance of both dimensions of analogy. On her construal of the 'meaning variance' problem, however, it became impossible to make headway on the logical problem. I have suggested that Hesse might have adopted the Kuhnian image of science, which allows for periods of normal science in which meanings are relatively stable.[18] On this picture, there is room for the view that the sciences are both a limiting special case and a leading special case for a theory of analogy. We draw on our understanding of metaphor to understand how meanings and connections shift as an analogy evolves over time, but we can also develop frameworks for evaluating analogical arguments based on argument forms commonly found in the sciences. Many challenges remain. How does an analogy evolve over time? What happens when a bad analogy becomes ingrained in scientific practice?[19] Tension between the logical and semantic accounts of analogy can be a fruitful source of problems for investigation.

Contemporary discussions of analogy face much the same tension. Given the difficulty of finding helpful criteria, let alone a universal logic, for evaluating analogical arguments, it is tempting to drop the logical problem altogether. My suggestion, following the lead of Hesse, is that we turn instead to mathematics and the natural sciences, where we find relatively stable forms

[18] As noted earlier, if Kuhnian normal science appears too rigid, Lakatosian research programs provide an alternative picture of scientific development with adequately stable concepts.

[19] See Stepan 1996 for discussion of an important historical example involving analogies between race and gender. See also Keller's paper in this volume.

of analogical argument. Tension between computational theories and logical approaches can be a fruitful source of problems, rather than a reason to abandon either approach.

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Paul Bartha
Vancouver Campus
Department of Philosophy
paul.bartha@ubc.ca